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THE MILANKOVITCH ASTRONOMICAL THEORY OF ALEOCLIMATES: A MODERN REVIEW

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1. INTRODUCTION

When climatic variations and variability have to be explained, it is fundamental to first clearly state the scale one would like to consider, both in time and space. Indeed, climate does fluctuate significantly from one year to another, but also it gradually changes in such a way as to make one decade, one century or one millennium different from the one before (Berger, 1979a). It is in fact because our memories tend to be too short to recall past years that we are alarmed when an unusually severe winter (1976-1977 in the U.S.A.) or prolonged drought occurs (1976 in Western Europe and Sahel).

Even if the longer-term records are still being laboriously reconstructed, there is much geo-ecological evidence that climate has fluctuated in the more distant past. This is particularly true for the last Ice Age. Figure 1 shows indeed that this ice age displays quite a lot of climatic variations, but even more particularly striking is the fact that these natural changes are essentially characterized by their respective amplitude : the mean rate of temperature variation over very long extended periods is of the order of 10° C/10,000 years, while over the historical period, it reaches around 1.5° C/100 years; since 1940, the mean rate of cooling, which is still in progress, is 0.02° C/year.

These yearly fluctuations and longer-term changes have been the result of natural processes at work on the complex system which determines the Earth's climate : atmosphere, oceans, cryosphere, lithosphere and biosphere. It is only since a few years that mankind appears to have become another significant factor in the climatic balance (Berger, 1980). In this present review, we will only focus on climatic variations with characteristic periods of 10,000 to 100,000 years which mainly represent glacial and interglacial oscillations during the Quaternary period. From a typical spectrum of climatic changes (Fig. 2) extending from periods comparable to the age of the Earth to about 1 hr, these Quaternary ice-volume cycles seem to be related to only one theory, the Milankovitch astronomical theory of paleoclimates.

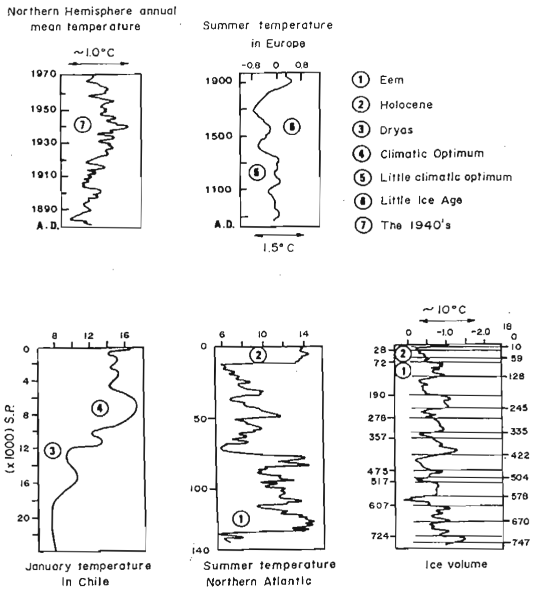

Fig. 1. Climatic changes during the Quaternary ice age (adapted from U.S. Federal Council for Science and Technology, 1974; paleoclimate detailed information available in Berger, 1979a).

2. ASTRONOMICAL ELEMENTS OF THE EARTH'S ORBIT

The astronomical theory of paleoclimates relates the climatic variations to those in the solar energy which would be available at the Earth's surface for a completely transparent atmosphere. For any latitude on the Earth, this insolation (Berger, 1975) is a single-valued function of the solar constant S_0, of the semi-major axis a of the ecliptic, of its eccentricity e, of its obliquity ε and of the longitude $\tilde{\omega}$ of the perihelion measured from the moving vernal equinox (Fig. 3).

The secular perturbations used in ordinary astronomical practice to compute these orbital elements of the Earth (expansion in powers of the time) are not appropriate for our problem, because they are adequate only for some centuries centered on the present. If information over longer intervals of time is desired, an analytical solution expressed in trigonometrical form is required, a solution the accuracy of which depends essentially upon the accuracy and the number of terms kept in the perturbation function of the Lagrange and Laplace equations.

Analysis of the differential equations of the planetary motion shows that those trigonometrical solutions progress in powers of the following small parameters: (1) the ratio of the masses of the planets to that of the sun, and (2) their eccentricities and inclinations to the reference plane. There are thus two kinds of approximation. The first involves the disturbing masses and the second involves the planetary es and is. In this way, solutions are dependent to the first or to the second order on the masses, and to the first or to the third degree on the planetary es and is if, respectively, terms of order equal to and higher than two or three in masses and terms of degree equal to and higher than three or five in es and is are neglected (even-degree terms do not exist, because the expansion of the perturbation function of the planetary system does not include odd powers of the es and is).

In fact, these solutions are obtained from assumptions made to solve the celestial mechanics fundamental equations in eccentricity e, longitude π of the perihelion measured from the fixed vernal point, inclination i of the orbit on the reference plane, and longitude Ω of the ascending node. As mathematical techniques for solving differential equations allow more and more sophisticated numerical computations, the assumptions need to be less and less restrictive, leading to solutions with a larger and larger number of terms. As the quality of observations is also improving, allowing a better determination of the constants and of the initial conditions, these techniques ought to provide more and more accurate solutions.

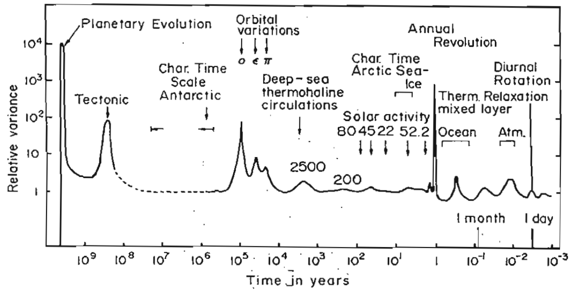

Fig. 2. Tentative spectrum of climatic variations. Estimate of relative variance of climate over all periods of the variation. A background level of variability, deriving from internal stochastic mechanisms and corresponding to a low degree of predictability, appears to increase in amplitude towards the longer time scales and to be over-topped by band-limited variability due to external forcing processes and corresponding to a high degree of predictability (adapted from Mitchell, 1976).

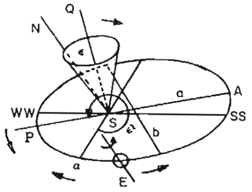

Fig. 3. Elements of the Earth's orbit. The orbit of the Earth, *E*, around the Sun, S, is represented by the ellipse *P*γ*EA*, *P* being the perihelion and *A* the aphelion. Its eccentricity, *e*, is given by $(a^2-b^2)^{1/2}/a$, *a* being the semi-major axis and *b* the semi-minor axis. WW and SS are respectively the winter and the summer solstice, γ is the vernal equinox; WW, SS and γ are located where they are today. SQ is perpendicular to the ecliptic and the obliquity, ε, is the inclination of the equator upon the ecliptic – *i.e.* ε is equal to the angle between the Earth's axis of rotation SN and SQ. $\tilde{\omega}$ is the longitude of the perihelion relative to the moving vernal equinox, and is equal to Π+Ψ. The annual general precession in longitude, Ψ, describes the absolute motion of γ along the Earth's orbit relative to the fixed stars. Π, the longitude of the perihelion, is measured from the reference vernal equinox of 1950 A.D. and describes the absolute motion of the perihelion relative to the fixed stars. For any numerical value of $\tilde{\omega}$, 180° is subtracted for a practical purpose: observations are made from the Earth, and the Sun is considered as revolving around the Earth.

A first-order solution for the eight principal planets was already given by Stockwell* more than 100 years ago. This Lagrange type of solution is obtained by the very drastic process of disregarding the periodic terms in the development of the disturbing function and limiting this disturbing function to the quadratic terms in the *e*s and *i*s (R_{02}). The system of differential equations for the elements of the principal planets then breaks up into two separate systems, one for $e\cos(\pi)$, $e\sin(\pi)$ and one for $\sin(i)\cos(\Omega)$, $\sin(i)\sin(\Omega)$. The expressions are linear homogeneous differential equations of the first order with constant coefficients, a determinant equation furnishing the periods of the individual terms.

In 1950, the improvement brought about by Brouwer and van Woerkom was attained through the use of more accurate planetary masses (Newcomb) and the evaluation of a second-order effect produced by the great inequality in the motions of Jupiter and Saturn. Indeed, Hill showed that the terms in the disturbing function for Jupiter and Saturn that arise from the great inequality $2\,1_J - 5\,1_S$ have a considerable influence on the secular variations of the eccentricities and perihelion, an influence which is at least as important as the one coming from the long-period terms of degree three originating directly from the secular part of *R*. It is thus the reason why Brouwer-van Woerkom and Sharaf-Budnikova have considered the Lagrange solution with, for Jupiter-Saturn, the Hill-Leverrier disturbing function where terms up to the degree six in *e*s and *i*s are taken into account. These short-period terms generated long periods, of which Brouwer kept only the two most important ones. This inclusion caused an appreciable change in one of the terms in the eccentricity-perihelion (e,π) solution:

	Period	Coefficient
Stockwell	348,700 year	0.01634
Brouwer-van Woerkom	301,700 year	0.01834

Although this new representation of the long-term variations of the Earth's orbital elements, when compared to Stockwell, shows differences up to 30% in various terms, the character of the variation remains the same.

As the *e*s and *i*s are small, the neglected secular terms of the perturbing function whose degrees with respect to the *e*s and *i*s are higher than the second are also small as compared with those not neglected. At the same time, however, it is possible that a very small additional term in a differential equation may have a notable effect in the integral, especially if the time be sufficiently extended. Although Leverrier and Harzen were already taking into account these terms of higher order in the secular part of the disturbing function, it was Anolik *et al.* who, for the first time in 1969, systematically considered all long-period terms of the disturbing function up to the fourth degree in *e*s and *i*s for all the eight planets.

Thus, as important modifications of these developments may arise from commensurabilities among the periods of the principal planets, the effect of near-resonances must be included, and it was in 1974 that Bretagnon made a decisive improvement in the evaluation of *e*, π, *i*, Ω. The main characteristics of that solution are as follows. The elements of the Earth's orbit are referred to the mean ecliptic and the mean equinox of 1850.0. The initial elements at this epoch of reference are the most accurate available, and the most recent determination of the planetary masses (Kovalevsky, 1971) is used. Constants of integration have been carefully determined in such a way that the solution coincides with the elements at the time origin 1850.0 (a calculation which is not found in a similar work by Anolik *et al.*, 1969). About the disturbing function, in addition to all the long-period terms up to the fourth order in planetary *e*s and *i*s, Bretagnon has introduced some terms coming from the short-period part of the disturbing function. About the influence of these short-period terms to the second order as to the masses, terms limited to the third degree in *e*s and *i*s, which lead to a modification of the long-period frequencies by more than 10^{-3} "/year (around 50 for the only Jupiter-Saturn near-resonance), have been kept for all the planets. Among the long periods generated through these terms, only those whose amplitude is greater than 10^{-4} for the inner planets and 10^{-6} for the major ones have been considered. Among the short-period terms used, in particular those of order three of the great inequality in the motion of Jupiter and Saturn are introduced.

In addition to *e*, π, *i* and Ω, the paleoclimatic problem requires also the obliquity ε and the annual general precession in longitude ψ. For ε and ψ, Poisson equations have been solved through the Sharaf-Budnikova (1967) method, where the series expansions include terms up to the second degree in eccentricity. This finally allowed Berger (1976) to determine the long-term variations of the elements of the Earth's orbit as required in paleoclimatology: this solution for (e,π,i,Ω) includes terms to the second order of the disturbing masses, and to the third degree with respect to the planetary *e*s and *i*s; for ε and ψ, Sharaf's analytical expansions were used, but their numerical determination takes into account all the Bretagnon terms.

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*A full list of references is given in Berger (1976, 1977a).

Critical analysis of theories of the long-term variations of the elements of the Earth's orbit (Table 1) and their numerical comparison leads to the following conclusions (Berger, 1977a) regarding the influence of different terms on the accuracy of the expansions used:

(1) further improvement in planetary masses will not have significant influence;

(2) for the (e,π) system, terms depending upon the second order as to the disturbing masses are more important than ones coming from the third degree with respect to the planetary *es* and *is*;

(3) for the (i,Ω) system, the latter terms have highly significant influence, whereas additional terms in masses are negligible. The same conclusion can be drawn for (ε,ψ);

(4) the most up-to-date solution (Berger, 1978a) is close to the ideal one and probably can provide valuable information up to 5 million years. However, it is advisable to be cautious as regards the absolute accuracy of the results, inaccuracies in the frequencies producing an effect the importance of which becomes larger and larger as the time increases.

Table 1. Evolution in the Computation of Long-Term Variations of the Earth's Orbital Elements

Power as to the disturbing masses	Degree in planetary eccentricities and inclinations			
		1		3
1		Lagrange (1781)		Anolik *et al.* (1969)
		Laplace (1798)		Bretagnon 2 (1974)
		Pontécoulant-Le Verrier (1834)		
	1	Stockwell (1870)		
		Harzer (1895)		
	1	Milankovitch-Stockwell-Pilgrim (1920)		
	1	Milankovich-Miskovitch-Le Verrier (1941)		
		Anolik 1 (1969)		
		Bretagnon 1 (1972-1974)		
2		Brouwer-Van Woerkom (1950)		Bretagnon 3 (1974)
	2	Sharaf-Boudnikova (1969)	1	Berger 1 (1973-1977a)
	2	Vernekar (1972)	2	Berger 2 (1976)
			2	Berger 3 (1978a)

Contributions to the long-term variations of the Earth's orbital elements have been classified according to the accuracy of the series expansions used for the eccentricity and inclination systems (i.e. following the degree in planetary *es* and *is* and the power as to the disturbing masses), and also according to the degree of the series expansion for the obliquity and the annual general precession in longitude (number given in front of the author's name). The larger the numbers are, the larger is the number of terms kept in the series expansions and, thus, the higher is the accuracy. As an example, the upper two groups lead generally to a solution which is accurate only for 100,000 years from today. More detailed information is given in Berger (1977a) (Table updated from Berger (1978b)).

As S_0 has been taken as a constant, 1353 W/m^2 (1.95 cal/cm^2/min), and a has no purely secular part when the perturbations of the second order are included, only the long-term variations of e, ε and $\tilde{\omega}$ must be determined. This solution (labelled Berger 3 in Table 1) for the classical astro-insolation parameters can be written in a simple practical form, still providing excellent accuracy (the most important higher-order terms are included in the series expansions):

$$e = e_0 + \Sigma\, E_i \cos\,(\lambda_i t + \phi_i) \tag{1}$$

$$e \sin\tilde{\omega} = \Sigma P_i \sin\,(\alpha_i t + \beta_i) \tag{2}$$

$$\varepsilon = \bar{\varepsilon} + \Sigma A_i \cos\,(\gamma_i t + \delta_i). \tag{3}$$

All the constants in (1), (2) and (3) are available in Berger (1978a).

Numerical experiments conducted over the previous 5 million years show that e varies between 0.0005 and 0.0607 (present value 0.0167), with an average quasi-period of 95,000 years, and that ε varies between 22°2' and 24°30' (today's value 23°27'), with a quasi-period of 41,000 years. The revolution of the vernal point relative to the moving perihelion (climatic precession) has an average quasi-period of 21,700 years ($e \sin \tilde{\omega}$ is presently equal to 0.01635), whereas relative to the fixed perihelion of reference, this quasi-period is 25,700 years (astronomical precession of equinoxes).

The procedure used for obtaining long-range changes in the elements of planetary orbits involves a drastic simplification of the astronomical problem and the results obtained are admittedly of a limited accuracy. The most serious limitations of a practical nature arise from the uncertainties of the periods themselves. These are obtained as functions of the masses and the orbital elements of the planets. They would be modified by the inclusion of terms of higher order in masses and higher powers in *es* and *is*. If any one of the principal periods should need a correction of 1%, it is clear that the contribution due to this particular term would be a full period out of phase after 100 periods. The representation by these series thus becomes less and less reliable as to detail as the interval of time from the epoch increases. If an uncertainty of 1% in a period of 90,000 years is a reasonable estimate, then the representation by these series has lost all its meaning as to detail in 9×10^6 years. Moreover, due to the large influence of the Jupiter-Saturn near-resonance, the calculations must be repeated including: (1) not only long-period terms of fifth order (terms which would affect mostly Mercury, Venus, Earth and Mars), but also, (2) for all the planets, short-period terms of higher order in *es* and *is*, and eventually, (3) terms of the third order in masses, at least for the Jupiter-Saturn near-resonance.

However, qualitative and quantitative indications are also available to show that the results are nevertheless reliable:

(1) The same method has been used with good success (Brouwer, 1953) in the treatment of satellite systems in which the periods may be shorter by factors of about 10^4. Thus, 100 years in such a satellite system may correspond to a million years in the planetary system.

(2) The application of the Brouwer-van Woerkom solution to the motion of minor planets indicates a general reliability to about the third decimal place, but does not indicate anything concerning the interval of time for which the developments may be trusted.

(3) There is an excellent agreement between the mean long-term period of ε and that computed from a numerical expression derived from the last 2000 years of observation (Wittman, 1979).

To sum up, some improvements are now expected in the field of theoretical researches, in the techniques used to determine the planetary motion and in the number of terms kept in the expansions used. The actual representation of the elements of the Earth's orbit in the simple trigonometrical form may be considered to be reliable in detail to four significant figures for approximately 3 million years. Even if, on account of the uncertainty of the periods involved, the details of the representation become more and more uncertain, the farther the epoch to be considered is removed from the present, the general character of the representation remains the same even for very distant epochs.

Although the problem of cross-dating is even more complicated due to the existence of the same kind of timing uncertainty in geological data, the overall accuracy in paleoclimatology is now improving and an absolute chronology of paleoclimates over the last 3 million years may be attempted.

3. THE MILANKOVITCH THEORY*

Adhemar (1842) was the first to suggest that the prime mover of the ice ages might be variations in the way the Earth moves around the Sun. He based his theory on the fact that over long periods of time, variations occur in the direction of the Earth's axis.
Croll (1875) took Adhemar's ideas and developed them into a new astronomical theory of climate. He reasoned that a decrease in the amount of sunlight received during the winter favours the accumulation of snow, and that any small initial increase in the size of the area covered by snow would be amplified by the snowfields themselves (positive feedback). After having determined which astronomical factors control the amount of sunlight received during the winter, he concluded that the precession of the equinoxes must play a decisive role. He also showed that changes in the shape of the orbit determine how effective the precessional wobble is in changing the intensity of the seasons. Croll's first theory predicts that one hemisphere or the other will experience an ice age whenever two conditions occur simultaneously: a markedly elongated orbit and a winter solstice that occurs far from the Sun. Later on, Croll hypothesized that an ice age would be more likely to occur during periods when the axis is closer to vertical, for then the polar regions receive a smaller amount of heat.

As time went on, many geologists in Europe and America became more and more dissatisfied with Croll's theory, finding it at variance with new evidence that the last ice age had ended not 80,000 (according to his view) but 10,000 years ago. Moreover, theoretical arguments were advanced against the theory by meteorologists, who calculated that the variations in solar heating described by Croll were too small to have any noticeable effect on climate.

It was only during the first decades of the 20th century that Köppen and Wegener (1924), Spitaler (1943) and above all Milankovitch (1920, 1930, 1941) indicated that it is the diminution of heat during the summer half-year which is the decisive factor in glaciation. In fact, this theory requires that a northern high-latitude summer must be cold to prevent the winter snow from melting in such a way as to allow a positive value in the annual budget of snow and ice, and to initiate a positive feedback cooling over the Earth through a further extension of the snow cover and subsequent increase of the surface albedo. On the assumption of a perfectly transparent atmosphere, that hypothesis thus requires a minimum in the northern hemisphere summer insolation at high latitudes.

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*A very complete summary of historical works on astronomical theory is available in Imbrie and Imbrie (1978), and a detailed list of references is given in Berger (1978c).

In fact, a caloric half-year was introduced by Milankovitch, because the length of the astronomical seasons shows secular variation (the caloric summer is the half-year which comprises all the days of stronger radiation; the other one being the caloric winter). The mathematical definition of such intervals implies that glaciation will occur when:

(1) the longitude of the perihelion is such that the northern hemisphere summer begins at the aphelion (referring to Fig. 3, this means $\tilde{\omega} = 270^\circ$);

(2) the eccentricity is maximum, which means that the Earth-Sun distance at the aphelion will be the largest. This eccentricity affects the relative intensity and the duration of the seasons in the different hemispheres, but also the difference between maximum and minimum insolation received in the course of 1 year, a difference which can amount to as much as 30% for the most elliptical orbit. In fact, e and $\tilde{\omega}$ are mainly used through the climatic precession parameter $e\sin\tilde{\omega}$. This parameter plays an opposite role in both hemispheres, and is a measure of the difference in length between half-year astronomical seasons and of the difference between the Earth-Sun distance at both solstices. It is the most important factor because critical climatic parameters such as insolation at solstices and caloric season insolation require its numerical values (Berger, 1978b);

(3) obliquity is low, which means that the difference between summer and winter is weak and the latitudinal contrast is large.

Following all these requirements, not only would the summer temperatures in northern high latitudes be fresh enough to prevent snow and ice from melting, but also mild winters would allow a substantial evaporation in the intertropical zone and abundant snowfalls in temperate and polar latitudes, the humidity being supplied there by an intensified general circulation due to a maximum latitudinal "thermal" gradient.

Moreover, as the high-latitude caloric insolations are mainly dependent on ε, whereas those of low latitudes are essentially dependent on $e\sin\tilde{\omega}$, and as the ε-effect is the same in both hemispheres, whereas the $e\sin\tilde{\omega}$ effect is opposite, the nature itself of this model implies compensation of negative summer deviations by positive winter deviations and an antisymmetry between hemispheres which becomes minimal for all latitudes higher than 70°.

4. MODERN VERSION OF THE MILANKOVITCH THEORY

Till around 1973, this theory has been largely disputed because the discussions were based on fragmentary geological sedimentary records and on Milankovitch summer radiation curves at 65° north, the absolute accuracy of which was not demonstrated. Moreover, this theory was in conflict with some well-admitted observations, namely the quasi-simultaneity of glacial ages in both hemispheres.

Despite improvements in dating and in interpretation of the geological data in terms of paleoclimates by Emiliani in 1955, and the use of some other insolation curves by Broecker in 1966, the following four fundamental questions related to the Milankovitch theory were not answered yet:

(1) Are the long-term variations of the Earth's orbital elements and of the insolation reliable?

(2) Are the quasi-periodicities of the Earth's orbital elements significantly present in the geological records?

(3) Is there any significant correlation between insolation curves and geological data?

(4) Can these insolation changes have induced climatic changes of a magnitude similar to those which have been recorded in the geo-ecological data?

4.1. Spectrum of Astronomical and Geological Data

Although a new modern solution has been produced in 1973 by Berger, it was only in 1976 that Hays *et al.*, and Berger, demonstrated that the astronomical frequencies were significantly present in paleoclimatic data. Indeed, from a careful spectral analysis of records in various deep-sea cores, Hays *et al.* (1976) have shown that the following quasi-periods were statistically significant: 105,000, 41,000, 23,000, and 14,000 years. At the same time, Berger (1977b) was determining all parameters of equations (1) to (3) where the periods related to the most important terms were proving to be respectively: for the eccentricity, 413,000, 95,000, 123,000 and 100,000; for the obliquity, 41,000; and for the paleoclimatic precessional term, 23,700, 22,400, 18,980 and 19,160; for these three elements, secondary peaks appear to be located respectively around 50,000, 53,000 and 30,000, 15,000 and 56,000 years.

Similar periodicities have already been found in Quaternary geological data by Chappell in 1973 and, maybe in a less rigorous way, by Mann (1967) and Van den Heuvel (1966b), and even in older epochs by Bradley (1929), Van Houten (1964) and Bond and Stocklmayer (1967).

This is remarkable, even if the time scale used in these investigations differs from Hays *et al.* by about a factor of two. The explanation of this paradox is to be found in the relative value of the astronomical periodicities and in the dominant climatic periodicity which in all of the Hays *et al.* cores is the 100,000-year cycle, and not, as expected, the geological response to the 41,000-year obliquity and to the 21,000-year precession cycle as predicted by a linear version of the theory of orbital control. Indeed, the spectral peak identified by Chappell as due to precession and by Van den Heuvel as due to precession half-cycle are now to be understood as the effect of, respectively, the obliquity and a full precession cycle.

In a recent publication, Kominz (1978) has shown that records from deep-sea core V28-239 display spectral peak periodicities centered on 104,000, 92,300, 58,500, 52,200, 41,000, 30,000, 23,000 and 19,000 years. In order to be sure that this observation was not simply an artifact of visual curve matching, the coherency between the $\delta^{18}O$ records and the orbital variations has been determined by cross-spectral analysis. Significant peaks with a coherency greater than 0.40 are only related to precession and obliquity, with obliquity consistently leading the $\delta^{18}O$ record by about 10,000 years.

For the eccentricity, the situation is far more complicated. The most important term of a 412,000-year period, needing a time series record long enough to distinguish such a periodicity, has been found in geological records only quite recently (Kominz, 1978; Harrell and Briskin, 1978; Shackleton, 1978), even if it has been forecast as early as 1976 by Berger. Moreover, its interpretation, and that of peaks in the range of 100,000-year containing most of the climatic variance, is difficult. These quasi-periods may be related either to the eccentricity periods themselves, or to a beat effect due to the non-linear interaction between the two precession peaks (Wigley, 1976). The problem is still unsolved because we have to find how much variance of the climatic variations can be explained by the 100,000-year period which originates directly from the eccentricity and how much originates from the non-linear response of the climatic system to the precessional forcing (Birchfield and Weertman, 1978). Looking to the influence of e alone on the insolation, we can see that it acts only through the $(1-e^2)^{-1/2}$ factor in the total energy received by the Earth, all other astronomical variables contributing to a redistribution of the energy among latitudes. Although these variations are small ($\sim 0.1\%$), they are in the right direction according to the results of Hays *et al.*: they are positively correlated with the change in summer temperature and not in the opposite sense as required by Milankovitch, who stated that the influence of e was almost exclusively through the $e\sin\tilde{\omega}$ term.

However, the coherency peaks that may be ascribed to eccentricity (Kominz and Pisias, 1979) do not fall directly on the 400 and 100×10^3-year geological periodicities. This implies that these spectral peaks do not bear a direct linear relationship to the eccentricity of the Earth's orbit, and that the 26% of the variance record attributed to astronomical forcing is only attributed to precession (6%) and to obliquity (20%). If these orbital parameters form the only deterministic component of the climatic record in the frequency range being tested, then the remaining non-random variance must be stochastic in nature and can be explained by the stochastic model of Hasselman (1976), in which the $\delta^{18}O$ record is forced by a much higher frequency process corresponding to a period of less than 12,000 years.

Although this value of 26% is much weaker than the 80% found by Hays *et al.* (1976), assuming that all the variance in the peaks in the power spectrum of the climatic record could be attributed to astronomical forcing, and is even weaker than the 41% of Chin and Yevjevich (1973), who described climate change as an almost periodic stochastic process, it is much higher that the 10% suggested by Keer (1978). However, if mid-month insolation is used instead of these orbital elements or even instead of the Milankovitch caloric insolations (as in Birchfield and Weertman, 1978) to test the astronomical theory, a much higher value is found for the explained variance (up to 87% in Berger *et al.*, 1980).

4.2. Insolation and Sensitive Latitudes

Most of the scientists who used the Milankovitch astronomical theory to account for the glacial oscillations during the Quaternary based their conclusions on a matching between geological curves and summer insolation curves obtained by Milankovitch (1941), Van Woerkom (1953), Bernard (1962), or most recently by Vernekar (1972) and Berger (1978b).

The first criterion which has been used is a visual or statistical correspondence between minima and maxima of both curves (Brouwer, 1950; Jardetsky, 1961). Milankovitch summer radiation curves for 65°N have been used more frequently because of the more extensive nature of Pleistocene glaciation in the northern hemisphere. Among the most active partisans of the Milankovitch school are Emiliani-Geiss (1957), who proposed a theory of glaciation using insolation changes to start (not to end) glacial cycles, and Zeuner (1959), who suggested that the Scandinavian ice sheet had its origins at approximately that latitude.

Analysis of the idealized curve for temperatures of the surface waters of the Atlantic Ocean given by Emiliani led Broecker (1966) to suggest an astronomical-climatic curve based upon an original combination of precession, tilt and eccentricity. The overall effect of his weighting factors was to create a deep 18,000 YBP minimum, two important maxima at 120,000 and 80,000 YBP, and a long period of intermediate climate between 70,000 and 22,000 BP, where the 48,000 YBP prominent warm peak found in the Milankovitch curve is greatly depressed. Assuming the half-response time for the continental glaciers to be 3000 years, his ice-volume curve became consistent with observation. Following this idea, Broecker *et al.* (1968) used the 45°N insolation curve (where the precession effect is given more weight than is tilt effect) in such a way that the warm peak at 50,000 YBP is largely removed and a new peak appears at 106,000 YBP. These results clearly indicated that the last four sea-level high stands (122, 103, 82, 5000 YBP) correspond closely in time to the last four prominent warm peaks (127, 106, 82, 11,000 YBP) in the modified curve of summer insolation, not only in chronology but also in magnitude (Mesolella *et al.*, 1969). However, based on very accurate dating of core V12-122, Broecker and van Donck (1970) had to increase by 35% the absolute time scale adopted by Emiliani for deep-sea cores. They then proposed 45, 55 and 65°N summer insolation as a tool to explain the gradual glacial build-ups over periods averaging 90,000 years in length and terminated by deglaciations completed in less than one-tenth of this time. At the same time, Veeh and Chappell (1970) obtained for the last 230,000 years the same kind of correlation between sea-levels derived from raised coral reefs of New Guinea and summer insolation at 45°N.

These qualitative coincidences of the principal maxima and minima of both curves are, however, somewhat illusory because they are not based on purely objective analysis but most of the time merely on preconceived ideas. Ruddiman and McIntyre (1976) have proposed the insolation at 55°N as a guideline, and van den Heuvel (1966a) that at latitudes north of 70°N because there the asymmetry between northern and southern hemispheres is largely attenuated, due to the fact that in Arctic and Antartic regions, the insolation variations caused by changes of the obliquity are much larger than those caused by changes in the precessional parameter. Following the same idea, Evans (1972) built up an absolute time scale for the whole Pleistocene from insolation received both at 65°N and 65°S, making an allowance for latent heat, albedo and 4000 years time lag. This is in opposition to Fairbridge (1961), who has referred to the northern hemisphere middle latitudes (40-75°) as the sensitive latitudes to climatic change, since approximately 95% of the world's mountain glaciers are located at this position. Kukla's (1972) proposal is based on the season-to-season difference in the mean insolation received beyond the latitudinal belt lying between 25 and 75°N in the winter half-years of the northern hemisphere, the fastest increase of this quantity being supposed to indicate interglacials such as Eem and Holocene.

The models by Calder (1974) and Mason (1976) are even more complete. Calder supposed that a decline in summer sunshine at 50°N (47 cal/cm/day) below a certain level (17 cal/cm^2/day) allows the volume of glaciers and ice sheets to grow in simple proportion to the deficit, while summer sunshine above that level melts ice with a different proportionality. With a melting rate five times the freezing rate, a realistic curve for the most recent glaciation (the last 78,000 years) has been obtained and applied over the past 860,000 years. Mason has reworked some aspects of the Milankovitch model in terms of the variations in the amount of heat received each year north of 45°N, a variation which is about 1% of the total heat received by the polar cap. When ice cover developed during the period from 83,000 to 18,000 YBP, the integrated deficiency in such insolation amounted to some 556 cal/g of ice formed. From 18,000 to 6000 YBP (melting of the ice), the integrated surplus of heat received was 10^{25} cal, while the latent heat required for the known decrease in volume of ice was 3.2×10^{24} cal. Such a close agreement with the corresponding latent heats might not be a pure coincidence. This simple astronomical model shows that the northern hemisphere insolation minimum will be reached in about 10,000 years but bottoming-out before reaching conditions quite so extreme as those which prevailed at the height of the recent ice age.

However, more objective techniques of data analysis, such as multivariate regression, principal component analysis, and spectral analysis could be used to model the relationship between orbital elements and geological data. The following astro-climatic index (ACLIN) successfully simulates the timing within the precision limits of the radiometric dating and the relative amplitude of gross paleoclimatic indicators during the last 250,000 years (Kukla and Berger, 1979):

$$T = \left|\frac{\tilde{\omega} - 180}{90}\right| + \varepsilon' - 22 + 500\, e^2$$

where $\tilde{\omega}$ and e are orbital elements at time t, and ε' is the obliquity at a time t' separated by the angular measure of 90° in the longitude of perihelion (approximately 5000 - 6000 years). ACLIN predicts the interglacials at 6 and 122 × 10^3 YBP, the interstatials 28, 54, 80 and 101 × 10^3 YBP, and the cold maxima at around 18, 66, 89 and 111 × 10^3 YBP, with the last one less firmly expressed than the first two.

In addition to all these proofs, the remaining nagging doubt about the mechanism — the near synchronicity of northern and southern glaciations — has also disappeared (Gribbin, 1978). The evidence indeed confirms that both hemispheres undergo near-synchronous glaciations, with an indication that the south leads the north into and out of an ice age, at least for the 18,000 YBP case. Since summers in the northern hemisphere get "colder" when those in the southern become "warmer", this evidence was long taken as refuting the model. However, the present distribution of continents — southern polar continent and a land-locked northern polar sea — provides the clue to explain global ice-ages within the overall Milankovitch models because it requires cold northern summers and cold southern winters to initiate the spread of ice. Indeed, in qualitative terms, it is clear that this initiation in the northern hemisphere requires cool summers to prevent winter snow and ice from melting. On the other hand, in the southern hemisphere, there is scarcely any land at high latitudes which is not already covered by permanent ice caps. Snow which falls on the sea cannot remain even if summers are cool, and the only way to spread the covering of sea-ice is to have very severe winters in which large volumes of ocean water are frozen, when once again the increased albedo plays a part in hastening the development.

4.3. Monthly Insolation and Paleoclimates

It is not only the sensitive latitudes which play an important role in climate modelling, but also the critical season. In that way, some important conclusions may be drawn from the fact that the obliquity peak in geological records is smaller than the 100,000-year peak or, at least, less dominant than formerly thought (Bernard, 1962). By analysing the solar radiation available on the assumption of a perfectly transparent atmosphere (Berger, 1975), it can be deduced, on the one hand:

(1) the insolation received at the equinoxes and the differences between the length of the summer and of the winter astronomical seasons are functions only of the precessional parameter;

(2) at the solstices, this parameter has a larger influence than the obliquity.

On the other hand:

(1) during astronomical seasons, the insolation received at any latitude is a function of the obliquity ε, and

(2) at high latitudes, the deviations of the insolation from present-day values for the caloric seasons are mostly functions of ε.

Moreover, the Milankovitch caloric half-years mask the intra-annual variability and its variations, and the whole summer season is not necessarily the most sensitive period of the year to explain the advance and retreat of ice sheets and glaciers (Kukla, 1975). As a consequence, the simulation of the past climate will thus need the knowledge of the past daily or monthly insolation instead of, or in addition to, the Milankovitch caloric season insolation.

As an illustration, it could be very interesting to show first how the numerical values of this monthly insolation pattern (Berger, 1978a) compare with and complement the classical Milankovitch caloric insolation (Berger, 1978b). For example, 18,000 years ago when a glacial maximum occurred with an Arctic ice-cap extending to the Netherlands, central England and New York, the obliquity, the precessional parameter and the eccentricity, whose values were respectively 23°27', 0.00544 and 0.01945, lead to an annual insolation and a caloric summer insolation close to their present-day value. However, the monthly insolation anomalies (Fig. 4) for the 60-70°N belt amounted to -35 cal/cm^2/day in August-September and $+40$ cal/cm^2/day in April-May, which represent respectively 8 and 5% of the actual corresponding daily insolation. Similar comparisons made over the last 1,000,000 years show that the deviations of the caloric insolations from their present-day values are always below 5%, whereas the deviations for the monthly insolations sometimes reach 12% (Berger, 1979b).

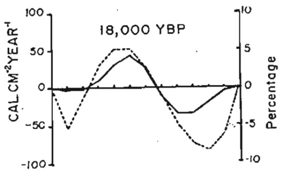

Fig. 4. Annual cycle of insolation 18,000 Y.B.P. Deviations, from their 1950 A.D. values, of zonal monthly mean for 60-70°N zonal belt. The full line is related to the left scale (cal/cm^2/d) and the dashed line to the right scale (%age of 1950 A.D. values).

Now, to figure out how the changes in the annual cycle from one 1000-year period to the following one are related to climatic variations, the dynamic behaviour of some insolation features must be analysed (Berger, 1979b). Many times during the last million years, a maximum in May shifts progressively towards July-August at the same time that a minimum appears in February and moves towards April. From time to time, this shift is faster, the deepening in the spring season is steeper, and is then rapidly replaced by a maximum, the spring minimum being shifted towards summer. This feature, called insolation signature, is thought to be related to a warm phase going into a cool one, a shortening of the lag times for the oceanic and cryospheric responses being expected due to the power of such insolation changes.

Over the last 500,000 years, such insolation signatures start around 505, 486, 465, 315, 290, 243, 220, 199, 127, 105, 84 and 13 thousand YBP. At these dates, a weak minimum is observed in late winter and early spring. A strong maximum culminates in June-July and is sometimes replaced by a deep minimum in some 7000 years. It is remarkable how well these dates fit with the maxima of the geological curve, namely that deduced from deep-sea core RC13-229 whose time scale has recently been revised, the boundaries from cold to warm main stages being now located 504, 422, 335, 245, 128 and 10 thousand YBP (Morley and Hays, 1979).

This analysis of the deviations of mid-month daily insolation, preferably from the mean state calculated over the last million years, clearly indicates that variations in the annual cycle of the insolation and in the amplitude of the monthly insolation are related to Quaternary climatic changes. It is not a linear real-time relationship, but it is their degree of steadiness in time which is thought to be responsible for the appearance or not of a full glacial or interglacial. Accordingly, the very flat pattern of the annual cycle observed at 60°N between 70,000 and 30,000 YBP must indicate the maintenance of the triggered glaciation around 75,000 YBP. The deep minimum starting in April around 30,000 YBP and moving towards June and July between 25,000 and 20,000 YBP must be responsible for the last glacial maximum that occurred around 20-18,000 YBP. When the speed of change of the annual cycle pattern from one 1000-years to the next is fast, going from a large maximum summer insolation to a deep minimum and back to a maximum in a very limited time span, a short cooling or "abortive" glaciation has to be expected. It is the case from 127,000 (maximum) to 118,000 YBP (minimum) and to 107,000 YBP (maximum again).

As far as the present Holocene interglacial is concerned, the 11,000 YBP July insolation maximum is clearly related to its beginning. From orbital geometry, it is also evident that the present-day insolation during summer months has been decreasing since 3000 YBP, will reach its minimum in July-August around 3000 YAP, and will not be significantly larger than the mean state before 24,000 YAP.

Finally, as at 94,000, 105,000 and 197,000 YBP, the main maximum deviation from the mean state is located at the south and not the north pole, as for the other relevant dates; it becomes evident that the northern hemisphere should not always be considered at the leading hemisphere. Consequently, the problem of lags and leads at the Quaternary time scale between the northern hemisphere and the southern hemisphere (CLIMAP, 1976,1978) can probably be partly resolved through the analysis of the mid-month insolation behaviour in the past.

4.4. Modelling the Dynamics of Climatic Changes

The Milankovitch model has thus passed both the test of physical plausibility and severe statistical tests, but there still remain difficulties in explaining how the relatively small changes in Milankovitch insolation could be sufficient to initiate or end glacial ages.

In models by Shaw and Donn (1968) and Sellers (1970) which, under astronomical forcing, generate climatic variations one order of magnitude too small, the albedo feedback mechanisms in high latitudes, the oceanic circulation, and the ocean-climate interactions (Barnett, 1978) were poorly simulated. If the seasonal extent of ice cover (Veeh and Chappell, 1970) and non-linear feedbacks related to the polar ice caps (Adam, 1975; Frederiksen, 1976), are included correctly, results begin to be much more significant (Johnson and McClure, 1976; Weertman, 1976). In this respect, from a zonally symmetric model of the global energy balance, incorporating the positive feedback due to the high albedo of snow and sea-ice and seasonally varying insolation, Suarez and Held (1979) get a response qualitatively similar to the geological record over the past 150,000 years when the model is forced with the orbital variations. Using a modified form of Weertman's model (1976), ice age continental ice-sheet growths and decays have also been simulated by Birchfield and Weertman (1978) from insolation anomaly data. Their model consistently predicts, in addition to significant responses at the 19,000, 23,000 and 41,000 years forcing periods, dominant long-period responses, most commonly at 100,000 and/or 400,000 years. This, of course, lends support to the hypothesis that non-linear response of the climate system to fluctuations of the orbital parameters is responsible for the long-period climate fluctuations recorded in deep-sea cores.

Moreover, a general circulation model with a shallow ocean (Mason, 1978) has showed that, on changing the Earth's orbital parameters from their present-day values to those 10,000 years ago when the Earth received 7% more solar radiation in June than at present, the atmosphere became warmer everywhere, with surface temperatures 6°C higher in the Arctic basin and 4°C higher at 30°N.

From all these results, there seems to be little doubt that the climatic system and the orbital elements are linked by a cause-and-effect relationship, although the precise linking mechanisms remains unknown. It is therefore meaningful to search for the percentage of the climatic variations which can be explained by the astronomical theory alone. This approach bypasses any solar-terrestrial link and assumes a direct relation (not necessarily linear) between monthly insolation and climate.

4.5. Insolation Climate Index

To objectively support and/or improve the correlations described in Section 4.3, multivariate analyses have been performed between $\delta^{18}O$ deep-sea cores data (Hays *et al.*, 1976) and the zonal monthly mean insolations. An appropriate homogeneous objective statistical selection led to monthly insolation for June 85°N and 55°S (J_{85} and J_{-55}), December 65°N and 75°S (D_{65} and D_{-75}), the intertropical latitudes being represented by March 25°N (M_{25}) and September 15°S (S_{-15}).

After different trials were performed (Berger *et al.*, 1980), two models gave particularly significant results. In the first one, based upon the spectral idea, warm and cool periods* have been dissociated to test if they are responding differently to the insolation forcing. For the cold period, (chosen as being 15-60,000 YBP), 67% of the total variance can be explained using only two principal components. This is significant at the 99% level and the serial-*R* of the residuals is not different from zero at the 90% level. For the warm periods (0-15,000 and 60-135,000 YBP), the explained variation is slightly less (still significant), but the serial-*R* of the residuals is definitely significantly different from zero. As all regression coefficients were highly significant, the reconstruction of past climatic variations has been made over the whole time-scale of available geological data (i.e. up to 500,000 YBP). Then, in contrast to the values mentioned in Section 4.1, this model reproduces as much as 50% of the total climate variations.

The other model is largely based upon the climatological meaning and implication of the insolation signature concept. It simulates the dynamic evolution in time of the climatic response to the insolation forcing, taking into account the memory of the climate system itself. In fact, persistence has been included and one of the predictors is now the climate as observed 3000 years before. Using four principal components to represent the seven input variables, the model equation becomes for standardized variables:

$$\delta^{18}O(t) = 0.924\ \delta^{18}O(t-3000) + 0.148\ J_{85} + 0.110\ D_{65}$$
$$+ 0.004\ M_{25} + 0.032\ S_{-15} - 0.036\ J_{-55} - 0.034\ D_{-75}.$$

Surprisingly enough, this relationship explains 87% of the total climatic variation, and the serial-*R* of the residuals is only 0.29, allowing extrapolation for the next 60,000 years.

The statistical value of both ACLIN and INCLIN series and their close agreement with geo-ecological paleoclimates authorize the prediction of the future natural climate. This, however, will only materialize if man's impact on land and the atmosphere has not yet modified the mechanism of climate change and does not do so in the future. The first cold peak will arrive 4000 YAP, and the models foresee an improvement peaking at about 15,000 YAP, followed by a cold interval centered around 23,000 YAP. Major glaciation, comparable to the stage four of the last glacial cycle, is indicated at 60,000 YAP (Fig. 5).

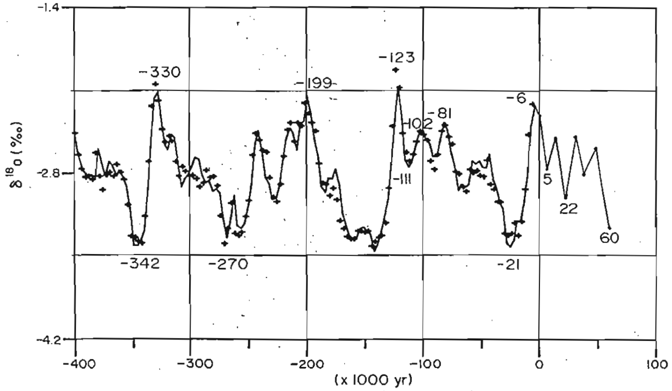

Fig. 5. Long-term climatic variations over the past 400,000 years, and prediction for the next 60,000 years. Crosses represent the $\delta^{18}O$ deep-sea cores data from Hays *et al.* (1976). Full line is the climate simulated by the auto-regressive insolation model. Extrapolation into the future is based on results from both ACLIN and INCLIN (auto-regressive) models: the high frequencies smoothed out INCLIN values for the future by the persistence predictor $\delta^{18}(t-3000)$, have been reconstructed from ACLIN results.

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*Only the last 135,000 years have been selected to calibrate the model, because of the excellent absolute accuracy of radiometric dating over this period.

All these results also strongly support the key role of some internal processes, like snow and sea-ice (Kukla, 1978), in the mechanism of climate change, and there is no doubt that the final explanation of Pleistocene climates, past as well as future, will have to include interactions of orbital perturbations with non-periodic terrestrial phenomena.

5. CONCLUSIONS

Assuming paleogeographical configurations and the solar constant being what they were at the beginning of the Quaternary, recent models, both qualitative and quantitative, conclude that the orbital parameters have modulated the climate and will continue to do so assuming no human interferences (Berger, 1980). As we have very accurate values for orbital elements and monthly insolations, we must now design both simple models which are able to reproduce the dynamic behaviour of climatic changes and variability, and more sophisticated ones, which allow us to test the validity of the simple models for selected particularly significant dates such as 122,000, 18,000 and 6000 YBP. Experience already shows that, without any doubt, these models will have to include seasonal variability of insolation, climate-ocean interactions and albedo-temperature-precipitation feedbacks.

Because of this success of the astronomical theory of paleoclimates on the Earth, similar studies have been undertaken for Mars (Ward, 1979), where the astronomical effect is even larger due to the planet's orbital properties and where the monthly insolation model will be applied.

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explained using only two principal components. This is significant at the 99% level and the serial-*R* of the residuals is not different from zero at the 90% level. For the warm periods (0-15,000 and 60-135,000 YBP), the explained variation is slightly less (still significant), but the serial-*R* of the residuals is definitely significantly different from zero. As all regression coefficients were highly significant, the reconstruction of past climatic variations has been made over the whole time-scale of available geological data (i.e. up to 500,000 YBP). Then, in contrast to the values mentioned in Section 4.1, this model reproduces as much as 50% of the total climate variations.

The other model is largely based upon the climatological meaning and implication of the insolation signature concept. It simulates the dynamic evolution in time of the climatic response to the insolation forcing, taking into account the memory of the climate system itself. In fact, persistence has been included and one of the predictors is now the climate as observed 3000 years before. Using four principal components to represent the seven input variables, the model equation becomes for standardized variables:

$$\delta^{18}0(t) = 0.924\ \delta^{18}0(t\text{-}3000) + 0.148\ J_{85} + 0.110\ D_{65}$$
$$+ 0.004\ M_{25} + 0.032\ S_{-15} - 0.036\ J_{-55} - 0.034\ D_{-75}.$$

Surprisingly enough, this relationship explains 87% of the total climatic variation, and the serial-*R* of the residuals is only 0.29, allowing extrapolation for the next 60,000 years.

The statistical value of both ACLIN and INCLIN series and their close agreement with geo-ecological paleoclimates authorize the prediction of the future natural climate. This, however, will only materialize if man's impact on land and the atmosphere has not yet modified the mechanism of climate change and does not do so in the future. The first cold peak will arrive 4000 YAP, and the models foresee an improvement peaking at about 15,000 YAP, followed by a cold interval centered around 23,000 YAP. Major glaciation, comparable to the stage four of the last glacial cycle, is indicated at 60,000 YAP (Fig. 5).

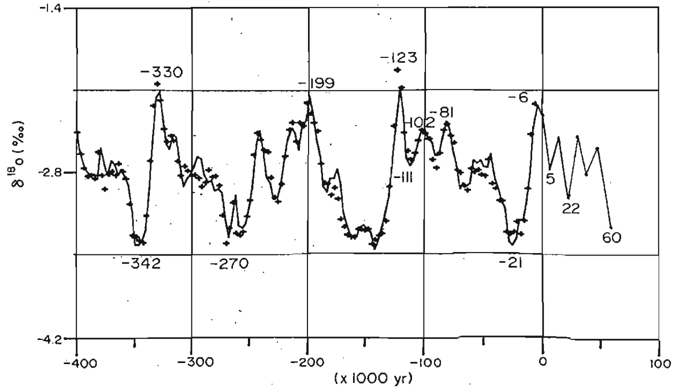

Fig. 5. Long-term climatic variations over the past 400,000 years, and prediction for the next 60,000 years. Crosses represent the $\delta^{18}O$ deep-sea cores data from Hays *et al.* (1976). Full line is the climate simulated by the auto-regressive insolation model. Extrapolation into the future is based on results from both ACLIN and INCLIN (auto-regressive) models: the high frequencies smoothed out INCLIN values for the future by the persistence predictor $\delta^{18}(t\text{-}3000)$, have been reconstructed from ACLIN results.

All these results also strongly support the key role of some internal processes, like snow and sea-ice (Kukla, 1978), in the mechanism of climate change, and there is no doubt that the final explanation of Pleistocene climates, past as well as future, will have to include interactions of orbital perturbations with non-periodic terrestrial phenomena.

5. CONCLUSIONS

Assuming paleogeographical configurations and the solar constant being what they were at the beginning of the Quaternary, recent models, both qualitative and quantitative, conclude that the orbital parameters have modulated the climate and will continue to do so assuming no human interferences (Berger, 1980). As we have very accurate values for orbital elements and monthly insolations, we must now design both simple models which are able to reproduce the dynamic behaviour of climatic changes and variability, and more sophisticated ones, which allow us to test the validity of the simple models for selected particularly significant dates such as 122,000, 18,000 and 6000 YBP. Experience already shows that, without any doubt, these models will have to include seasonal variability of insolation, climate-ocean interactions and albedo-temperature-precipitation feedbacks.

Because of this success of the astronomical theory of paleoclimates on the Earth, similar studies have been undertaken for Mars (Ward, 1979), where the astronomical effect is even larger due to the planet's orbital properties and where the monthly insolation model will be applied.

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