Revisiting the microfield formulation using Continuous Time Random Walk Theory

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Outline

Microfield Model Method
The Continuous Time Random Walk Theory and its application to Stark Broadening
The Kangaroo Hypothesis
Time Memory effect
Application to Ly-a
References

Semi-classical theory for line profile

$$i\hbar \frac{d}{dt} U(t,t') = (H_0 + V(t) + B) U(t,t')$$

$$V(t) = -\vec{D}.\vec{E}(t)$$

H_0 – Hamiltonian operator for the unperturbed atom

$V(t)$ – time-depending perturbing potential due to the motions of ions

B – operator of electron impacts

Assumptions:

- dipole approximation for the perturbing potential
- perturbers cause only radiative transitions
- perturbers are classical uncorrelated particals moving in straight lines

Microfield Model Method

$$\frac{d}{dt}U(t,t') = M\left(\vec{E}(t)\right)U(t,t')$$

Linear stochastic equation of the evolution operator

E(t) – fluctuating microfield

$$P\left(\vec{E}\right)$$

$$\Gamma(t) = \left\langle \vec{E}(t).\vec{E}(0) \right\rangle$$

Probability distribution and covariance of the microfield ⟶ crucial role for Stark profiles

U. Frisch and A. Brissaud, J. Quant. Spectrosc. Radiat. Transfer 11, 1753 (1971)

C. Stehlé, Astron. Astrophys. Suppl. Ser. 104, 509 (1994)

B. Talin et al., Phys. Rev. A 51, 4917 (1995)

This work

-To revise the **Stochastic Processes** used to model the plasma microfield, namely the Kangaroo Process, developped by *Frisch and Brissaud* (1971), at the light of the Continuous Time Random Walk (CTRW) of *Montroll, Weiss* (1965)

-Use the CTRW for calculating the ion dynamics effect on the Lyman alpha line

-Explore possible improvements of the stochastic process

E. Montroll and J. Weiss, J. of Mathematical Physics 6, 167 (1965)

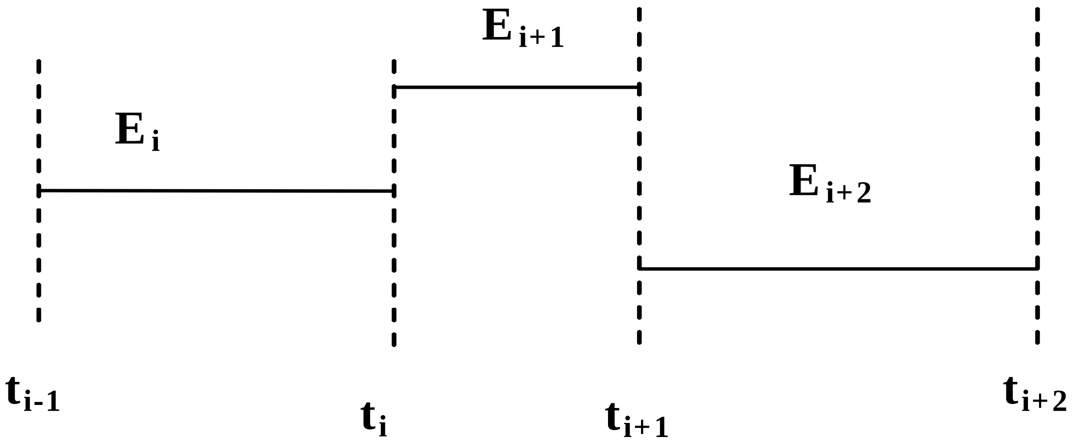
Continuous Time Random Walk (CTRW)

Elliott Montroll (1916-1983)

CTRW: the time interval between two jumps in the stochastic variable obeys a given probalility law (Waiting Time Distribution)
Kangaroo Process is a particular case of CTRW process using the *Kangaroo Hypothesis* and a *Poisson* Waiting Time Distribution

Consider **E**(t) as a stochastic vector

$$E(t) \rightarrow \{E_i, t_{i-1}\}$$



Continuous Time Random Walk (CTRW)

Key Quantity: *probability density*

$$\psi_{(t_i - t_{i-1})}(E_{i+1}, E_i)$$

Stationary process

Statistically independent jumps

$$\int d^3E \int_0^\infty dt\, \psi_t(E, E') = 1$$

$$\int d^3E'\, \psi_t(E', E) = \varphi_E(t)$$

Waiting Time Distribution

$$\Phi_E(t) = 1 - \int_O^t dt'\, \varphi_E(t')$$

Probability that no jumps occur during t

Solution of the Schrödinger equation for a stepwise constant process

$$dU(t,t_0)/dt = -i(H_0 - D.E(t))U(t,t_0)$$

$$U(t,t_0) = U_{E_{m+1}}(t - t_m)U_{E_m}(t_m - t_{m-1})\ldots U_{E_1}(t_1,t_0)$$

Single field history of an atom during t

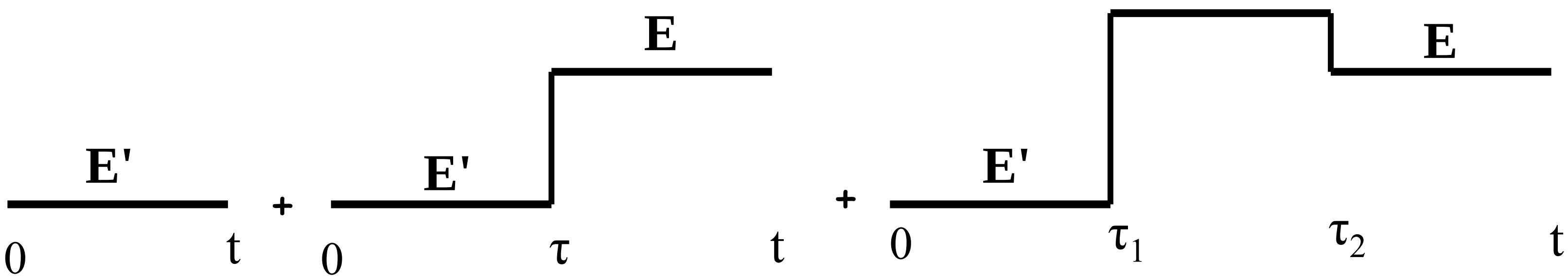
$$U_{E_k}(t_k - t_{k-1}) = \exp{-i(H_0 - d.E_k)(t_k - t_{k-1})}$$

Time evolution for an atom in a static field E_k

Averaging of the evolution operator over electric fields

1) Calculate the atomic time evolution operator for all possible **E** field time histories with the initial value **E'** and final value **E** at time t $T_t(E,E')$

$T_t =$



$$T_t(E,E') = \delta(E-E')\Phi_E(t)U_E(t) + \int_0^t d\tau \int dE_1\, T_{t-\tau}(E,E_1)\, U_{E'}(\tau)\, \Psi_\tau(E_1,E')$$

Averaging of the evolution operator over electric fields

2) Average over all initial and final electric fields

Electric field Distribution

Hooper

$$T(t) = \int d^3E \int d^3E' \, P(E') \, T_t\,(E,E')$$

New part: Generalization of Kangaroo Process

Kangaroo Hypothesis → $\psi_t(E,E') = q(E)\varphi_{E'}(t)$

q(E) - the probability density to find an electric field **E** after one jump

Waiting Time Distribution $\varphi_E(t)$ – any function

New part:
Generalization of Kangaroo Process

Exact solution for the atomic evolution operator

$$\tilde{T}(s) = \int d^3E\, P(E)\tilde{Y}_E(s) + \int d^3E\, q(E)\tilde{Y}_E(s)\left[I - \int d^3E\, q(E)\tilde{X}_E(s)\right]^{-1} \int d^3E\, P(E)\tilde{X}_E(s)$$

$$\tilde{X}_E(s) = \int_0^\infty dt\varphi_E(t)\exp-\left[s + \frac{1}{i\hbar}(H_0 - \vec{D}.\vec{E} + B)\right]t$$

$$\tilde{Y}_E(s) = \int_0^\infty dt\Phi_E(t)\exp-\left[s + \frac{1}{i\hbar}(H_0 - \vec{D}.\vec{E} + B)\right]t$$

Standard Kangaroo process

$$\psi_t(E,E') = q(E)\varphi_{E'}(t)$$

$$\varphi_E(t) = \nu(E)\exp - \nu(E)t$$

In diagonal representation

$$p = \left[s + \frac{1}{i\hbar}(H_0 - \vec{D}.\vec{E} + B) \right]$$

$$\tilde{X}_E(s) = \tilde{\varphi}_E(p)$$

$$\tilde{Y}_E(s) = \tilde{\Phi}_E(p)$$

The Frisch-Brissaud's result for the atomic evolution operator T(t) is recovered if the Waiting Time Distribution φ_E (t) is a Poisson's one

U. Frisch and A. Brissaud, J. Quant. Spectrosc. Radiat. Transfer 11, 1753 (1971)

Main difference from Frisch-Brissaud's solution

$$(\partial/\partial t + iH_0 + id.E)T_t(E,E_1) = -\int_0^t K_E(t-\tau)T_\tau(E,E_1)d\tau + q(E)\int_0^t d\tau \int d^3E' K_{E'}(t-\tau)T_\tau(E',E_1)$$

Memory kernel

$$\tilde{K}_E(s) = p\tilde{\varphi}_E(p)/(1-\tilde{\varphi}_E(p))$$

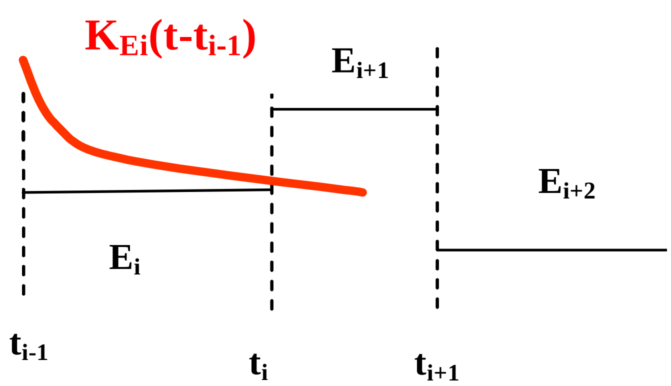
$$p = \left[s + \frac{1}{i\hbar}(H_0 - \vec{D}.\vec{E} + B) \right]$$

Poisson WTD

$K_E(t) = \nu(E)\delta(t)$ ⇒ $\tilde{K}_E(s) = \nu$ = const

Arbitrary WTD

$K_E(t)$ non-local in time

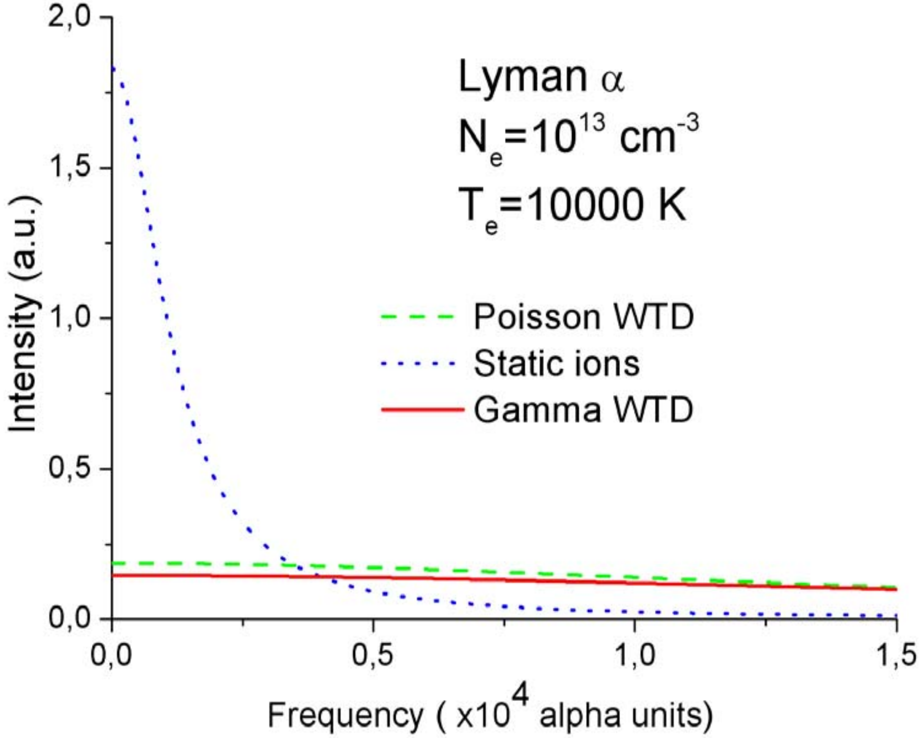


jumps with different electric fields overlap in time

Application to Ly-a line

Low density

(near impact)



Effect of the electric field dynamics

$q(E)=P(E)$

$$\varphi_E(t)=\frac{\nu}{\Gamma(a)}\,(\nu t)^{a-1}\,\exp{-\nu t}$$

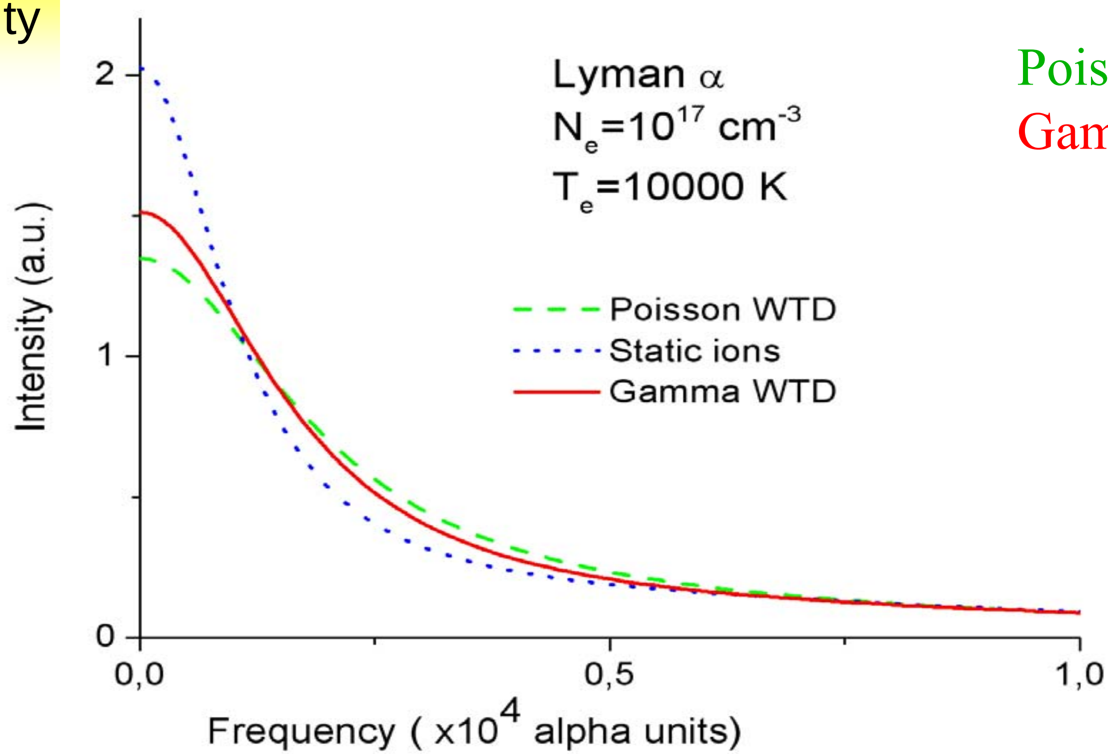
WTD

Poisson (a = 1)

Gamma (a = 2)

Application to Ly-a line

High density
(near static)



Poisson WTD a = 1
Gamma WTD a = 2

Line profile with Gamma Waiting Time Distribution is close to the static profile

Conclusions

- The Continuous Time Random Walk allows to generalize the Frisch Brissaud result to arbitrary Waiting Time Distribution

- The CTRW solution retains memory effects

- Preliminary applications to Ly-a show a significant influence of the choice of the Waiting Time Distribution

Perspectives

Stark Broadening
-Comparison with simulation results
-Use of Levy WTD for turbulent electric fields

Collisional Radiative Model with fluctuating plasmas parameters

Neutral Transport in turbulent plasmas

References

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