THE CONCEPT OF LOGIC

by

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In order to clarify the meaning of the term "logic" and to establish criteria which various calculi and theories must satisfy in order to belong to logic, a higher-level theory in relation to semantics (a "general theory of logic") is constructed. Logic is defined as the class of theories consisting of expressions which are (i) publicly meaningful, (ii) provable, and (iii) applicable to a certain lower-level theory T.

A logical expression is publicly meaningful if it can be interpreted in terms of some natural language. It is provable if it can be formulated as a axiom or as a consequence in a consistent formal system. It is applicable if it can be interpreted as a truth-condition for T.

When T is the whole of science, the meta-theory which lays down its truth-conditions is a general logic. When T is a special scientific theory, its meta-theory is a special logic.

There is no absolute distinction between empirical and logical truth. A universal empirically-true statement can be transformed into a logical statement of the corresponding special logic, and conversely, some logical formulae when applied can be transformed into universal empirically-true

statements.

As the conditions of empirical truth comprise rules of meaning, rules of proof and rules of verification, a logical theory L should contain semantics, theory of proof and theory of verification for T.

A meta-logical theory lays down the conditions of logical truth for L. These are interpretability, provability and applicability, therefore meta-logic should contain semantics, theory of proof and theory of application for L.

There is no cleavage between formal and non-formal logic if the rules of the former should be applicable and the intuitive procedures of the latter explicitly stated. What remains is the difference between general and special logic, which is relative to the field of application.

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Introduction

A characteristic feature of the most developed sciences (such as mathematics and physics) during the last century has been the great interest taken in the problems of their foundations. An immense effort in that direction has resulted in the relative clarification of the fundamental concepts of these sciences, such as number, space, time etc. Logicians have contributed much to this achievement, and conversely, work on the problems of science has made them realise how narrow and sterile the classical formal logic was. It would not be an exaggeration to say that the modern renaissance of logic is due largely to the awakened interest of logicians in the problems of foundations of mathematics and the general methodology of empirical science.

However the fundamental concepts of logic itself are not sufficiently clarified - and this is what always happens in the periods of quick growth which have been preceded by a long stagnation. Even a superficial survey of what has been done in logic since the time of Boole and de Morgan leads to the conclusion that, on the whole, logicians have been more interested in constructing new logical systems and removing difficulties of a technical nature than in interpreting these systems and examining their underlying implicit assumptions.

The consequence of this one-sidedness is a rather chaotic situation in the field which one might call "a general theory of logic", whose task should be the clarification of concepts such as meaning, truth, logic, meta-logic etc. All these concepts are either extremely vague or clarified only for a certain restricted field (as Tarski succeeded in the definition of truth for the class-calculus of symbolic logic). For instance logicians who belong to different philosophical schools not only disagree as to the explicit definition of what is logic but even hold incompatible views as to whether a particular work belongs or does not belong to logic. There is a chasm between the views that logic is concerned with the stages of the development of Absolute Spirit and that logic is merely an arbitrary symbolic calculus satisfying solely certain formal requirements. And while the latter has been regarded by many eminent philosophers as a kind of intellectual gymnastics having nothing to do with science, the former is interpreted by the majority of modern logicians as metaphysics, having nothing to do with logic.

A theory which deals with the nature and definition of meaning, truth and logic is a higher-level theory in relation to semantics. If a formal calculus is an object language and the semantics a meta-language, then the general theory of logic where the concepts of meaning, truth and logic are

clarified and truth-conditions for the sentences of semantics are provided, must be expressed in a meta-meta-language.

A difficulty in building up such a general theory of logic is that if it is to provide a criterion for deciding which formal calculuses and semantic systems belong to logic and which do not, it cannot presuppose any particular formal system of logic. (Otherwise we should get a vicious circle.) Only those symbolic forms (terms, procedures of construction etc.) can be taken for granted which underlie all rational discourse in modern science and philosophy. In other words, we assume the minimum of an informal logic which is generally accepted and implicitly given in all logical systems and scientific methodology.

So, for instance, we presuppose three kinds of undefined terms: first, a number of the terms of ordinary language, including connectives such as "if ... then", "or", "is" etc.; second, epistemological terms such as "experience", "symbol", "other people", "operation", "instrument", "practically relevant" etc.; third, some general logical terms whose meaning is intuitively given such as "individual", "class", "relation", etc.

The number of these undefined terms could be considerably decreased if the formal definitions of some were given in terms of the others. However, it would take too much space to accomplish this task and besides, it is irrelevant for our present purpose.

The undefined terms of the general theory of logic might be defined in a theory of a still higher level - perhaps a general philosophical theory of symbolic forms (including those used in the arts, moral life, myths, ordinary language etc.).

An insight into the meaning of the terms "meaning", "truth" and "logic" can be obtained only by a critical investigation of the already existing theories in which an attempt to clarify these concepts is made. If such an investigation is to be critical, there must be a certain criterion of appraisal, which has been established step by step in the process of inquiry, taking into account the advantages and difficulties of various theories. Once acquired in the process of our investigation, this criterion ought to be explicitly stated in the exposition of its results.

The criterion by which I justify the various conceptions of logic involves the following assumptions:[1]

(i) There is already a verified body of knowledge - science, and there are various logical systems. The logical systems which are really significant and generally accepted as belonging to logic stand in a particular functional relationship to science.

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1. It should be stated again that these assumptions are results of enquiry which appear as the postulates only in the process of exposition.

(ii) This relationship can be best described by saying that logic provides truth-conditions for a particular theory or a special science or for science in general. These truth-conditions are schemes which in various ways regulate our establishment of the truth of certain sentences (rules for the clarification of meaning, schemes of proof, principles of verification etc.).

(iii) Whether the claim of a system of symbols to be considered as logic is justified or not, depends on whether the schemes it provides really can be interpreted and used as the rules for establishing truth (i.e. for clarification of concepts, derivation of (factually) true sentences from (factually) true sentences, etc.).

(iv) Whether a meta-logical conception of logic, of truth and of meaning can be accepted or not depends on whether it succeeds in accounting for the actual functioning of the already existing and accepted logical systems, which all satisfy the condition described in (iii).

Chapter I

Main Modern Conceptions of Logic

There is an enormous number of different definitions of logic in the text-books and treatises belonging to different philosophical trends and schools. It is not necessary to investigate each of them in particular. We shall enumerate only some main types which are still important and influential, without bothering about their particular authors.

1. Classical conception of formal logic: Logic is the science which investigates the formal rules of thought.[1] The representatives of such a view are the vast majority of formal logicians from Aristotle up to the middle of the nineteenth century.

2. Ontological conception: The subject of logic is the study of the general structure of objective reality.

When the stress is laid on the independence of reality from thought (although this independence does not imply that

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1. "Formal logic may be defined as the science which investigates regulative principles of thought that have universal validity whatever may be the particular objects about which we are thinking." (T.V. Keynes, Studies and Exercises in Formal Logic, London, 1894, p.9.)

the structure of reality is not expressed in the structure of some particular system of thought) we are dealing with realism.[1]

When a distinction is made between individual thought and Thought as an objective all-pervasive and unifying entity, which is the basis and creative force of all reality, we have before us an objective idealism as in Hegel.[2]

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1. "Logic deals with the real world as zoology does, although it deals with its more abstract and general characteristics." (Russell, Introduction to Mathematical Philosophy, 1920, p.165.)

2. "Logic is the science of the pure Idea: pure, that is, because the Idea is in the abstract medium of Thought. Logic might have been defined as the science of thought and of its laws and characteristic forms. But thought as thought, constitutes only the general medium, or qualifying circumstance, which renders the Idea distinctively logical. If we identify the Idea with thought, thought must not be taken in the sense of a method or form but in the sense of the self-developing system of its laws and constituent elements. These laws are the work of thought itself and not a fact which it finds and must submit to."
(The Logic of Hegel, translated from the Encyclopedia by Wallace. Oxford, 1874, p.25.)

3. Epistemological, aprioristic conception: Logic is the theory of knowledge which investigates in the first place "a priori" forms, on the basis of which we construct our knowledge of the world (our concepts of objects etc.). This view was advocated by Kant and the New-Kantian school.[1.]

4. Phenomenological conception: Logic deals with the structure of pure ideal essences, norms which hold independently of any material existence on the one hand, and any thinking on the other. This view was held by Husserl[2.] and the members of the phenomenological school.

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1. "The logic can be taken in hand for two objects, either as Logic of the general or of a particular use of the understanding. The former contains all necessary rules of thought without which the understanding cannot be used at all. It treats of the understanding without any regard to the different objects to which it may be directed. Logic of the particular use of the understanding contains rules how to think correctly on certain classes of objects."
"General logic is either pure or applied .....General but pure logic has to deal with principles "a priori" only, and it is a canon of the understanding and of reason, though with reference to its formal application only, irrespective of any contents whether empirical or transcendental..... The former alone is a real science." (Immanuel Kant, Critique of Pure Reason, trans. by Max Müller, London, 1922, pp.41-43.).

2. "We have taken the concept of logic, following its historical tradition, as the science of Logos in form of Science, or as science of the essences that make a genuine science as such." (E. Husserl, Formale und transzendentale Logik, Halle, 1929, S.24.).

5. Psychologistic approach: Logic is concerned with the abstract structure of the operations of rational thinking. Such was the view of many nineteenth century empiricists and logicians who wanted to show that psychology is the basis of logic.[1.]

6. Pragmatico-instrumentalist approach: This conception is connected with the preceding one in so far as thinking is interpreted as a kind of activity. However the emphasis is laid upon the practical aspect of this activity: logic is concerned with it in so far as it serves to transform our surroundings in accordance with some practical purpose. The subject matter of logic is, then, the methodology of inquiry, and the logical forms are only the instruments of inquiry.[2.]

7. Nominalistic conception of logical positivism: Logic is any interpreted formal calculus which exhibits some structure

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1. "Logic, then is the science of the operations of understanding which are subservient to the estimation of evidence: both the process itself of proceeding from known truth, to unknown, and all intellectual operations auxiliary to this." (J.S. Mill, A System of Logic, vol.I. Introduction, London 1843, p.1.

2. "The subject-matter of logic is determined operationally..... The methods of inquiry are operations performed or to be performed. Logical forms are conditions that inquiry qua inquiry has to meet..." (Dewey, Logic, The Theory of Inquiry, New York, 1938, pp.11-15.

of symbols. The logic of science is the calculus which can be interpreted as the language of science.[1.]

This enumeration, which is not exhaustive, may make it seem that the concept of logic is very vague; otherwise such enormous difference in the views which are taken of it would not have been possible. Thus, a hegelian might say that a treatise on symbolic logic is only a game with symbols, and a modern formal logician would certainly wish to reply that Hegel's "Science of Logic" is only a bad ontology.

However a more careful examination of these different conceptions of logic might show that if certain underlying implicit assumptions were taken into account and if the

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1. "Philosophy is to be replaced by the logic of science - that is to say by the logical analysis of the concepts and sentences of the sciences, for the logic of science is nothing other than the logical syntax of the language of science...."

   ".....By the logical syntax of a language we mean the formal theory of the linguistic forms of that language - the systematic statement of the formal rules which govern it together with the development of the consequences which follow from these rules."

   "....A theory, a rule, a definition, or the like is to be called <u>formal</u> when no reference is made in it either to the meaning of the symbols (for example, the words) or to the sense of the expressions (e.g. the sentences) but simply and solely to the kinds and order of the symbols from which the expressions are constructed."
   (R. Carnap, <u>The Logical Syntax of Language</u>, London, 1937, p.xiii, 1.)

ambiguity of many expressions were avoided by suitable translations, many of the differences would disappear and the relation between various conceptions would turn out to be more one of overlapping and disjunction than of mutual exclusion.[1]

In such a situation the best possible procedure is:

1. To try to discover at least some common constituents of all these conceptions. These will provide a criterion - although necessarily a weak one - for excluding everything which does not belong to logic.

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1. Thus, for example, there is no reason why all logical systems could not be expressed in the form of a symbolic language. There would be two kinds of differences among various systems - differences in structure and differences in interpretation. Taken separately each of these two kinds of differences might suffice to make two systems incompatible, but, when interpreted this incompatibility might disappear. Such is the case with Hegel's famous rejection of the principle of non-contradiction. In a calculus of his logic the formula $(x)(fx \cdot \sim fx)$ would be true. If x were interpreted as any constant object or any concept with a fixed intension and extension the resulting proposition function would be obviously false for all values of x. However, Hegel's semantics would be very different. The meaning of x= would have to be "any object y which satisfies at least one of the following three conditions: (i) At the beginning of a time-interval t, y has a certain property and at the end of t, y has it no longer; (ii) y is a borderline case between two classes, only one of them having a certain property; (iii) y has a certain property only in relation to some objects and situations (in a certain context).

With these semantic qualifications the formula $(x)(fx \cdot \sim fx)$ would be true. All terms satisfying conditions (i) - (iii) could be substituted for x and as a result we should obtain in all cases empirically true propositions. Obviously such a logic is in no way incompatible with the ordinary formal logic which deals with constant and sharply limited objects.

2. In opposition to this weakest possible criterion for deciding the range of the applicability of the term "logic", to find out which among the given criteria is the strongest. In this way we will get the range of the vagueness of the term.

3. We shall try to suggest how this range of vagueness may be diminished by establishing a criterion stronger than the weakest one and weaker than the strongest one. If this criterion is precise enough, the suggested concept of logic will have quite fixed boundaries. If it is more flexible, these boundaries will still be blurred, although to a much smaller extent than before.

Now if we try to take step (1) we must answer the question: what do the different conceptions of logic that we have enumerated have in common?

Many modern logicians would not admit that logic in any way refers to reality or that it tells us anything about the world. Some of them would even deny that it is a science - it is only a language.

What might be generally accepted is only that the term "logic" refers to the investigation of some intersubjective structure of entities of a very general kind. "Intersubjective" here means: something which can be publicly known, which is

independent of any particular private experience and thinking. "Structure" is an abbreviation for a system of constant relations among ~~things~~ elements of a certain class. "Entities" here are either mere linguistic signs, or concepts, propositions, and principles, or things and classes of things, essences, or operations of thinking or practical actions of inquiry etc.

Now all logicians would agree that in their theories they are concerned with any of the entities mentioned only in so far as they are expressed in some language. The constituents of what a realist says are not properties and relations of material objects themselves, but some verbal signs which presumably designate them. In a similar way, not thinking as a mental process but "formulated thinking" (as Beno Erdmann put it), thinking expressed in a language, is obviously the only thing which can be investigated in a general way in logic. It is true that some philosophers use also the term "logic" in another sense when they speak of "the logic of things" and "the objective logic of reality" meaning by that the system of most general objective laws of nature. However, the most we can do is to postulate that there is something to which our "logic of things" refers - namely a certain natural order which is independent of human beings. Whatever we <u>know</u> about this natural order must take the form of propositions, and these are inseparable from

the linguistic form in which they are objectified and rendered communicable.

Therefore, in searching for the weakest possible criterion of logic we can temporarily reduce all entities to "meaningful linguistic symbols". The class of meaningful linguistic symbols contains as elements all terms which can be met in any logical theory.

So, when we say "logic is a set of theories which investigate intersubjective structures of meaningful (interpreted) symbols", although most logicians would object that logic is much more than that, all of them would probably admit, or it would be compatible with the actual import of their logical doctrine to say, that logic is at least that, and that whatever does not satisfy the given criterion should not be considered logic by anyone.

This is the weakest criterion of what is logic. It coincides to a great extent with a nominalist conception of logic. What a nominalist might wish to put more precisely is only that the structure mentioned must be a calculus, i.e. must contain the rules of the formation and transformation of the symbols.

The adherents of all other schools and trends in logic would suggest that a system of signs must satisfy a number of other conditions in order to be admitted as logic. For example, these signs might be interpreted as concepts and judgements and

the rules of the language as the rules of valid reasoning. Or, they might be given an operational interpretation. Or again, it may be argued that the structure of the symbols in a logical language reflects the general structure of reality, etc.

The last one seems to be stronger than any other possible criterion of logic, because it establishes firstly, a connection between language and thought, and secondly, between thought and objective reality. In such a way all those systems of symbols that do not express the structure of valid thinking are rejected, and as validity and truth are not the same, only those systems of valid thinking are accepted that correspond to the most general features of the material world or to some hypostatised objectively existing "thoughts". In modern logic this view was supported by Frege, the young Russell, Gödel, etc.

Frege laid very much emphasis on the assertion that linguistic signs must always stand for some objects and that the nature of the objects and their properties and relations determines the nature of rules for operating with symbols. So he wrote:

> "A logically perfect language (Begriffsschrift) should satisfy the conditions that every expression grammatically well constructed as a proper name out of signs already introduced, shall in fact designate an object, and that no sign shall be introduced as a proper name without being secured a reference."[1]

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1. Frege, Selections from Grundgesetze, trans. by Geach and Black Oxford, 1952. p.70.

A limit in this direction would be represented by the view that reality is a closed system and that its general structure comes to its full and absolutely accurate expression in a particular logical system, so that anybody who thought in accordance with the principles of such a logical system would be able to know the absolute truth about the world. All the other systems, if logical at all, would be false. Such a view was expressed in Hegel's logic.

Now we have got the two boundaries of the extension of the term "logic". The wider one reduces logic to a particular language, which need not necessarily tell us anything about the world. The narrower one lays down much stronger conditions presupposing not only the existence of mental entities such as concepts, judgements etc., but also the existence of reality and the relation of identity between valid thinking and the world.

The first asks too little, the second too much, and our next task is to explain why neither of them can be accepted.

For the first radically nominalist conception of logic, Carnap's famous principle of tolerance is characteristic:

> "Everybody is at liberty to build up his own logic, i.e. his own form of language. All that is required of him is, that, if he wishes to discuss it, he must state his methods clearly and give syntactical rules instead of philosophical arguments."[1.]

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1. Carnap, Die Logische Syntax der Sprache, Wien 1934, S.44-46, London, 1937, pp.51-52

Although Carnap has substantially changed his conception of logic in the last twenty years and although he himself no longer thinks that the construction of a system of logic is a matter of mere convention,[1] there are still many logicians who are not prepared to follow his modifications and who are very critical of his new and rather realistic view.

Conventionalism in logic is very far from being generally abandoned, even by Carnap himself, and therefore it deserves a detailed discussion.

The first objection to conventionalism is that just because its criteria of what logic is are so weak it leads us to a confusion of what practically everybody would accept as logic with what hardly anybody would.

It is well known that people interested in chess have invented a peculiar language which is used on the one hand for the symbolic representation of games which have been actually played, on the other hand for the theoretical analysis of possible games. It would be rather easy to give this language the form of a calculus K.

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1. "The principle of tolerance (perhaps better called 'principle of conventionality', as explained in [Syntax] § 17 is still maintained. It states that the construction of a calculus and the choice of its particular features are a matter of convention. On the other hand, the construction of a system of logic, i.e. the definitions for the L - concepts within a given semantical system is not a matter of mere convention; here the choice is essentially limited if the concepts are to be adequate." R.Carnap, Introduction to Semantics, Cambridge, Massachusetts, Harvard Univ.Press, 1942, p.247.

We should have firstly to give the classification of signs in K. This would consist of (a) six signs, one for each kind of piece, (b) sixtyfour signs for each particular field of the chess-board, (c) signs for the number of any move, (d) the two signs for check and mate, (e) sign for the transformation of a pawn into some other piece, (f) and (g) signs for full stop and commas. (sign for the castle)

Rules of formations in K would describe conditions under which a combination of the given signs would be considered as the expression of a move; e.g. a number followed by the sign of a piece, followed by the sign of the square at which the piece arrived, followed by the sign of a piece, followed by a comma, followed by another sign of ~~the square at which the~~ a piece (black piece this time), followed by another sign of the square at which the piece arrived, followed by a full stop. For the sign of a piece together with the sign of a square one of the two signs of castle may be substituted, etc.

The rules of deduction usually contain primitive sentences (axioms) and rules of inference. These primitive sentences would describe the initial positions of all pieces.

Rules of inferences would be in this case the linguistic expression of the rules of the game, i.e. rules which prescribe how each piece should move from one square to the other, what happens to the pawns when they reach the eighth row, when the game is finished, etc.

All this would constitute the syntax of the chess language. There would be no difficulty in building up the semantics of the language.

If we choose English as the meta-language, then we shall be able to explain what is designated by the signs of the calculus K (rules of designation) and also to lay down rules of truth by saying that any expression E, which designates a move, fg. "12.P - K5, B -R5" is a true expression of the language if the figure n at the beginning really is the nth move of the game and if white and black pieces designated by the sign of E really arrived at the places designated by the sign of E.

Now, is such a language a logical system? A conventionalist would be tempted to answer in the affirmative. This language is not only constructed in the form of a calculus but also has its interpretation. It contains criteria of truth so that we can derive true expressions from the true expressions.

But, however, what a queer logic it would be. We do not find in it any of the well known logical constants. There are no classes, predicates and relations in the ordinary sense. The system does not contain any of the well known rules of deduction. There are no negative expressionsand consequently no principles of contradiction and excluded middle in the given language.

However some very formalistically-minded philosopher might still wish to call our language logic. "This is the logic of chess", he might say, "and its peculiarity is due just to the fact that it is the logic of chess and not of science or of our everyday language."

But, then someone could go further and invent a completely arbitrary calculus whose rules of formation and deduction have no resemblance whatsoever to any known calculus. For his meta-language he might also invent some artificial language which nobody understands. He might claim that the signs of his calculus are meaningful because he has laid down the rules of designation. His system contains also semantic rules of truth in which the names of sentences of his calculus are quoted and then the translations of sentences in terms of his artificial meta-language are given.

Suppose that nobody understands his language. Suppose that even its creator does not understand it. He wanted simply to play a game, and to invent a structure of symbols, such that although the formal conditions of a calculus are satisfied, its structure has no rule in common with any other language. He continued the same game with his semantic. Suppose that for the term "is" he took the symbol ϕ for the term "true" the symbol $\Subset$, and for the term "when" the symbol $\wr$. Now when he wanted to express the truth condition of some sentence from his syntax he took the name of the sentence, fg.$\odot$ correlated it with some invented sign fg.$\circledcirc$ and then stipulated that $\circledcirc$

is the metalogical translation of ⊙ . In such a way he got the rule " '⊙'ϕ⋹ϰ⊙."

Now our friend, although frankly admitting that he has not the slightest idea what his signs correspond to in ordinary life or science, claims that, nevertheless, he knows how to derive other true expressions from them.

If he is only using an odd notation there is no objection in principle to that. However, our assumption is that there is also a profound difference in structure between his "logic" and the ordinary logic.[1.] In other words his odd language expresses an odd way of thinking and, let us assume, most of his truths would be generally considered as false statements. For example, he would throw dice, and if he got a six, that would mean that an arbitrarily constructed statement was true, otherwise it was false. In this way he would select his initial statements. From them he would get all the rest of his "tautologies" by applying strange rules, as, for example, the rule for making universal statements out of particular ones. (i.e. "$(\exists x)fx \supset (x)fx$").

The author of this queer system could not and would not say that what he has constructed is a logic of anything, as there was some sense in calling our formalised language of

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1. By this term is meant (a) informal logic of ordinary life, (b) wellknown and generally accepted logical systems which have already found their application in philosophy, mathematics and empirical sciences.

chess "logic of chess". He might argue that he is not obliged to construct a logic of anything, and nevertheless, on the basis of the definition of logic which he has found in many text-books on symbolic logic, what he has invented is a perfectly sound logic.

To which it should be replied that (i) unless he gives some explanation in the ordinary language, his symbolic structure is objectively meaningless; (ii) even if his language were comprehensible, what is expressed by it would not be logic because it would make us accept false statements for true.

As to (i) we cannot understand his language because we are given no key for translation. Usually we understand the meanings of symbols either (a) by reading explicit semantical definitions or (b) by observing how the symbols are used. Here semantical definitions do not help us because the terms in which they are given need to be defined themselves. Neither can we conclude anything about the use of symbols because this presupposes a similarity in structure between the two languages. For instance, the man might have in his logic a rule of deduction such that from the truth of a disjunction and of one of the constituents the truth of the other constituent follows. If we could translate the rule into ordinary symbolic language we should get the expression:

$$\frac{\vdash p \vee q \qquad \vdash p}{\vdash q}$$

Then we should be tempted to think that his sign for (let us say "Δ") is a sign for "$\subset$". This would mislead us and only after some time would we eventually discover that he does not use "Δ" the way we use "$\subset$". What should we do then? We have tried the rule we have, now if we want to investigate whether perhaps his notation, which is different from ours, expresses a wrong rule, there is such an enormous number of possibilities that it would be useless to try them. Each deviation from the structure of ordinary logic makes our re-interpretation more and more difficult.

As to (ii) we would certainly not accept as logic a system of rules which would constantly mislead us. The rules which allow us to infer from "Some people are bad" that "All people are bad", which justify us in drawing a false conclusion, "The earth is flat" from the true premisses "The earth is round or it is flat" and "The earth is round" are certainly not rules of logic from an objective point of view.

Our man may think in accordance with his own particular logic, but we should certainly agree that his thinking is illogical. But what right do we have to think that our logic is better than his? We feel that in the case mentioned we have such a right, but there is nothing in the conventionalist thesis to justify this right. Conversely, the essence of conventionalism is to give equal right to everybody who satisfies certain purely formal criteria.

Of course, if logic is nothing more than this, every man with some education in modern logic might invent a new logical system at his leisure. These useless and perfectly insignificant and even incomprehensible formal systems would have to be considered to be as logically respectable as the significant ones.

If one remembers what was historically the starting-point of the positivist movement, then the possibility of such a development seems slightly paradoxical. Because, the point of the principle of tolerance and the whole undertaking of the formalisation of logic - at least on the part of logical positivism - was the elimination of meaningless metaphysical assumptions. Now the whole of logic would become meaningless in another, but even worse sense. Because, as Professor John Wisdom once pointed out, metaphysicians somehow understand each other. At any rate within the framework of a certain school or trend, terms have at least some loose meaning. Now the principle of tolerance allowes everybody to have his own logic without bothering even whether anybody understands it. The limit is that even the author of the system does not necessarily need to understand what it means in ordinary language and what bearing it has on any knowledge or experience. Such a freedom of choice, had it been really used, would have led to complete confusion as to what is logically correct reasoning and what is

not. To say for a proposition that it is "logically true" would not have meant anything because every proposition might have been made by someone to be "logically true" by constructing rules like the above mentioned. Consequently, logic as science would have become completely redundant.

However, the important thing is that logicians refuse to exercise their freedom and almost always take the same meanings for symbols, the same constants, axioms, rules of deduction etc. Their systems are all very similar to each other, and even when some apparently radical changes are introduced as e.g. many-valued logic, they seem much more to be the result of some sophistication than a complete break with one type of logic and transition to some quite different type.

Logicians, even those who proclaim an absolute freedom in their creative work, seem to be bound by other considerations than the too loose formalistic criteria which they acknowledge as their own guiding principles. And as according to these guiding principles all implicit assumptions in logic should be made explicit, the same has to be applied also to the theory about logic.

And this brings us to the second objection to conventionalism: conventionalism does not pay enough attention to the fact that what even formalistically-minded logicians (and in a similar way mathematicians) really do when they construct a formal calculus

is not to invent a purely artificial structure of symbols but in the first place to look for the formalisation of an already given, or at least possible, theory. In other words, they try to build up such a structure of symbols as represents the conceptual structure of a certain given or possible theory, (its form). We may take as examples Hilbert's axiomatisation of geometry, Zermelo's axiomatisation of the set-theory, Birkhof's and Neuman's formalisation of the theory of quanta, Reichenbach's and Carnap's formalisation of the theory of probability, etc. This is the simplest case of the relation between a formal calculus and some theoretical knowledge. The former is the abstract picture of the latter, its form, the skeleton of which the latter is the body.

In the more complicated cases, a formal calculus is constructed such that some theory or science as a whole can be derived from it. In this case the purpose of the calculus C_1 is not to represent the form of the corresponding theory T_1, but to construct a formulation for the theory T_2 whose form is represented in the calculus C_2. In this case C_1 may deviate considerably from T_1 which amounts to the same as if we said: T_1 had to be modified and adjusted for the role of foundation-theory for T_2.

A clear example of this kind of procedure is given in the work of Frege, Russell and Whitehead. They have introduced

considerable modifications in existing logical theory in order to construct a logical system such that mathematics can be derived from it. They have been free to some extent in choosing notation, the basic logical constants (whether to define implication in terms of disjunction or vice versa) and even in the choice of some of the axioms, but their freedom was very much limited by the goal they wanted to attain. To say that the premises of their system were purely arbitrary stipulations is wrong and very misleading.

The third kind of cases comprises those where the relation between a calculus and some theory is even more complicated. Here C_1 is neither the form of some T_1 nor the foundation of some C_2 which is the form of T_2. What is here the aim of the logician may be roughly described in this way: There is a class of facts F which are not yet incorporated in any theory. The logician constructs a calculus C such that there are reasons to believe that C might represent either the form of a possible theory T about F or the foundation for the corresponding calculus of T.

For example, Lukasiewicz was led to the construction of the three-valued logic by reflections about the undecidability of statements about future events. Some of these statements are true, some are false, but for a large number of them we simply cannot decide about their truth value, and express this hesitation

by saying that they refer to some possible states of affairs. There was no direct application for any of the many-valued logics, but however, there were many cases which indicated their possible applicability. The fact that in spite of the application of many-valued logic in the theory of probability, it is still condemned by some ardent adherents of two-valued logic as unfruitful and artificial does not only show how some prejudices can be deeply rooted, but also how paradoxical are some discrepancies in the assumptions of many contemporary logicians. Sometimes it is the same people who are too liberal when they treat the premises of a logical system as arbitrary stipulations and too conservative when someone modifies any of the customarily accepted premises.

Finally we come to the borderline cases. I shall enumerate some of them.

(a) Someone constructs (or undertakes some important steps towards the construction of) a seemingly artificial calculus. Nobody thinks it has anything to do with logic. A few hundred years later this system finds important applications in science and philosophy and becomes generally accepted as a new logic. To this sort of case belong Leibniz's ideas about "Characteristica universalis" more than two hundred years before Principia Mathematica.

(b) Somebody invents a calculus which is incurably sterile but which is used as a prototype for the construction of a fruitful logical system. Much of the work which has been done in "algebra

of logic" belong to this sort of case.

(c) A calculus is constructed such that it has some similarity in structure with the well known logical systems (e.g. P.M.). However, for some reasons we are pretty certain that the calculus in question is purely artificial and cannot be applied as an abstract structure for thinking about any matters of fact or any philosophical, mathematical and scientific problem.

Such is the actual practice in the field of modern formal logic. It is clear that in the first three most important groups of cases <u>there is a functional relationship between a calculus and some theory or some unsystemised facts</u> and that relations between symbols in the former only belong to logic in so far as they represent the logical form of the latter. Here the procedure of a logician is arbitrary only in so far as he can: (i) choose one of several possible conceptual frameworks for the generalisation or foundation of the same facts and same theories; and (ii) choose one of several equivalent languages for the expression of the same conceptual framework.

For example, logistics, formalism and intuitionism represent three different conceptual frameworks for the foundation of the same science - mathematics - and one is free to choose any one of them. On the other hand Frege and Russell used two different languages to express a similar conceptual framework (the main difference being the incorporation of the theory of types in Russell's system).

In the borderline cases the freedom of a logician seems to be greater. In fact these are the only ones which can make the doctrine of conventionalism to some extent convincing. However some alternatives to a conventionalist explanation seems not only possible but also quite satisfactory. For for example it might be argued:

1. No science has fixed boundaries, not even mathematics. Why should it be different with logic? These borderline-cases belong to a range of vagueness in the extension of the term "logic", where we might hesitate to decide whether a calculus belongs to logic or not. So we might be induced to change our decision from negative to affirmative in the cases (a) and (b) on the basis of subsequently establishing a connection between our calculus and science. In the case (c) where we have every reason to believe that even the possibility of such a connection is excluded, we should be inclined not to regard the system in question as logic in spite of some apparent similarity with generally accepted systems.

2. Either it is the case that at a given moment t a structure of symbols can be used as a meta-theory which lays down the truth conditions for some theory, or it is not. In other words either it is the case that there is some functional relationship at t between a calculus and science, or it is not. What has happened at the moment $t - 1$ or at the moment $t + 1$

is irrelevant. We decide on the basis of our present knowledge without bothering about past and future. This is what we usually do in science whenever we decide that x is true or not. It does not matter what truth value had been assigned to x before we acquired some important information about it, or what we shall think about x in future on the basis of some subsequent knowledge. We neglect the former and we simply do not know anything about the latter.

In such a way our concept is sharply limited and we would not hesitate at the given moment to decide in the affirmative for (a) and (b) and in the negative for (c).

Both these explanations are possible. They are quite compatible. We get (2) from (1) when we simplify our problem by leaving aside the temporal dimension and by eliminating the possibility of our subjective uncertainty - taking into account only what is the objective state of affairs.

The advantage of the theories which give logic the status of a science interconnected with other sciences and distinguished from a game is that they give a better account than conventionalism of what logicians actually do. The main weakness of conventionalism was, we have seen, that it forced us to accept as logic symbolic structures which, as we strongly felt, had nothing to do with logic or science in general.

Now we come to investigate the opposite extreme to conventionalism. This is the view held by many realists,

objective idealists, ~~crude~~ uncritical materialists etc. that logic reflects reality in such a way that there is an identity of structure between our words and the concepts expressed by them on the one hand, and the outside world - be it the world of material things, essences or of some kind of pure objective ideas, norms etc. - on the other. A very modern and representative exposition of this view is found in Kurt Gödel's important article on Russell's mathematical logic.

"Classes and concepts may, however, also be conceived as real objects, namely classes as "pluralities of things" or as structures consisting of a plurality of things and concepts as the properties and relations of things existing independently of our definitions and constructions."

"It seems to me that the assumption of such objects is quite as legitimate as the assumption of physical bodies and there is quite as much reason to believe in their existence. They are in the same sense necessary to obtain a satisfactory system of mathematics as physical bodies are necessary for a satisfactory theory of our sense perceptions and/in both cases it is impossible to interpret the propositions one wants to assert about these entities as propositions about the "data", i.e. in the latter case the actually occurring sense perceptions. Russell himself concludes in the last chapter of his book on Meaning and Truth though "with hesitation", that there exist "universals", but

apparently he wants to confine this statement to concepts of sense perceptions which does not help the logician. I shall use the term "concept" in the sequel exclusively in this objective sense."[1].

Gödel's view is quite clearly a reaction to the difficulties which mathematical logic in general and Russell's in particular was involved in following nominalism. However, Russell's nominalism was a reaction to the difficulties implied by his own realistic views expressed in his early logical writings. So the pendulum swings from one side to the other, always tending to reach one of the extremes and then returning to the opposite one.

It is peculiar in Gödel's method of approach to the problem that he considers the identification of concepts with properties and relations of material things as "necessary in order to obtain a satisfactory system of mathematics." A doubt arises immediately Is it justifiable to accept a thesis with grave consequences for all sciences (because the categories in question are extremely general) only because it might be useful for mathematics? The history of philosophy should lead us to distrust philosophical generalisations which are built up on the basis of a single science.

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1. Kurt Gödel, "Russell's Mathematical Logic" The Philosophy of Bertrand Russell, The Library of Living Philosophers, vol.V, p.137. Evanston, Illinois.

The awkward consequences of the belief in the identity of concepts on the one hand and properties and relations of things on the other are at least these two:

1. If concepts are "conceived as the properties and relations of the things existing independently of our definitions and constructions" it is impossible to give a satisfactory account of the transformation of their content. For example, the concept of atom underwent a long evolution from Leucippus and Democritus to Rutherford and Bohr, and from Bohr to Schrödinger and Dirac. If we hold that at any particular time our concept of atom had to be "conceived" as a set of properties and relations of independently existing physical particles then either (i) we should maintain the absurd thesis that material things change in accordance with the transformations of our concepts, or (ii) we shall admit that it was a mistake to believe in the identity of our concepts and corresponding properties (and relations) of material things, however still holding that we are justified in doing so as to our present new concepts.

Realists usually accept the second alternative and allow for the discrepancies between our knowledge and things themselves in the past but not in the present. However, if it must in principle be allowed that after some time the present will become past, this double criterion hardly seems to be justifiable. The history of science and logic is the history of our concepts, besides other things. They changed their

extension or intension, and even in the most favourable cases we had to realise that they are not applicable in the sense we believed they were. Now, by the identification of concepts with the properties and relations of things, realism excludes the possibility of their being inapplicable or only imperfectly applicable to the experiential data. One should believe that false generalisations and arbitrary constructions belong only to the past. There are few things which are as incredible as that. In view of the whole of our experience with concepts, it would be much wiser to assume that, if they refer to properties and relations of things, they do so in a much more indirect and flexible way than is taken for granted by realists.

2. The first consequence of the realist thesis is a perfectly static conception of logic which is inconsistent with the fact that logic, like all other sciences, has its history in time and makes allowance for a certain plasticity of its forms.

3. The second consequence of the fundamental realist thesis is that at any given moment there is only one possible conceptual framework which might make a theory true or which a theory should possess in order to be true. The truth of a theory does not mean anything else but that the concepts used in it are identical with the properties and relations to which the theory refers, in which case any theory based on some different concepts must be false.

However much more flexibility is needed in order to account for what is actually going on in modern sciences. It is well known that very often scientists explain the same group of facts by several different theories which definitely use different concepts and disagree among themselves to some extent. Sometimes such alternative theories are more or less equally satisfactory from the point of view of their simplicity and ability to predict subsequent experiences. Take, for example, the famous issue between "corpuscular" and "wave" theory of light. Even nowadays when we think that there is a certain complementarity between these two aspects of light it is very difficult for us to imagine light having at the same time properties of undulating movement and consisting of particles of matter. This theory works and the most we can say is that there is a correspondence between its concepts and some properties of light – not that these concepts are the properties themselves or identical with them in their content.

In logic realism implies the necessity of choosing just one ideal logical system which pictures the general features of reality. This is the main obstacle to nominalism which at least theoretically allows an unlimited number of logical systems. However, whereas on this one may take any game-like structure of symbols with the rules of formation, transformation, designation

and "truth" cannot be accepted as logic, on the other hand it is impossible to accept only "Principia Mathematica" and reject the logic of Brouwer and Heyting. These are two different logical systems but they can both be used for the foundation of mathematics. Carnap's two-valued and Reichenbach's many-valued logic are two different logical systems but they can both be applied to the interpretation of the theory of probability. They do not exclude each other but at the same time they are not equivalent (from the point of view of their power of explanation) to any arbitrary system of symbols that might amuse a clever man to invent.

The other conceptions of logic that have been listed come between nominalism which provides the weakest criterion and realism which gives the strongest one. Phenomenology and Kantian apriorism suffer from the same lack of flexibility as realism. The assumption of "pure ideal essences" which do not exist either in mind or in objects and the assumption of the eternal skeleton of categories which constitute our reason, besides being rather mythical in character, also fail to give account of the dynamic character of logic and of the plurality of logical systems which both satisfy certain formal criteria and are more or less fruitful in their application in sciences.

In this sense we may say that all these absolutist tendencies are too dogmatic. What is claimed to be "essence of

reality" or "ideal norms" or the "a priori" forms of our reason" are only conceptual schemes of a single school or author, determined strongly by the situation in mathematics and other sciences at a given time and therefore more or less important for subsequent work on logic, but without any absolute value.

In opposition to this group of theories which are inclined towards absolutism and dogmatism there is another group which like conventionalism errs on the side of excessive flexibility. To this group belong all pragmatico-operational-instrumentalist conceptions of logic. Here the logical forms are conceived either as useful practical habits or useful instruments in the process of inquiry or sets of useful operations in our experimental activity. The consequence is a pluralist conception of truth. Too much stress on the practical aspect of thinking leads, in a consistent logical theory, to views similar to those which have been expressed in a very exaggerated, almost paradoxical way in James's work Pragmatism:[1.]

> "...An idea is "true" so long as to believe it is profitable to our lives." "....What would be better for us to believe? This sounds very like a definition of truth". "...If theological ideas prove to have a value for concrete life, they will be true for pragmatism in the ....sense of being good for so much."

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1. Pragmatism. London, 1907. p.75, p.72, p.73.

"....You can say either that (any idea) is useful, because it is true or that it is true because it is useful." "...Our account of truth is an account of truths in the plural."[1]

Pragmatism in the form presented to us by James, leads to the following paradox:

1. Let us suppose that we have three persons A, B and C who claim the truth of their statements P made by A, Q made by B and R made by C. Being pragmatists their reasons are: Each of these statements has been successful when applied in practice and has led to results satisfying their individual interests.

2. Now suppose that the statements P, Q, and R are mutually incompatible, expressing the incompatibility of A, B and C's interests and attitudes.

3. Each of the three individuals, suppose A, is in a paradoxical position, in so far as he follows the pragmatic criterion of truth. On the one hand he will hold that only P is true because only P has been shown to be a satisfactory guide in his practice. Q and R would lead him in a direction opposite to his interests, therefore they are false. But on the other hand, as a pragmatist who allows a plurality of truth, he is expected to admit that Q and R, being successful guides in other people's practice, are also true.

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1. James, Pragmatism, p. 204, quoted.

The root of this paradoxical in James's attempt to give a purely subjectivist and irrational attitude an objective theoretical form.

Even in its improved form in Dewey's instrumentalism, the pragmatic conception of logic suffers from too much "plasticity" which is the result of too much stress on practical activity as the criterion of truth. Dewey avoids the name of pragmatism but agrees that he is a pragmatist, if the term means that the validity of a statement depends on its consequences being established in an operational way so that they solve the specific problems which prompted the inquiry.[1] Therefore all logical forms are only instruments for reaching our particular vital goals in a well-controlled inquiry.[2]

The weak point in instrumentalism is that even if one assumes that being a logical system entails being an instrument of a successful theoretical and practical activity, the reverse does not necessarily hold. It is perfectly conceivable that one can be guided by some principles which are very hard to qualify as logical and in spite of that reach one's goals by pure chance. In general, as the truth of conclusions does not prove that the premises were not false, in a similar way the achievement of certain ends does not prove that the means were not badly chosen.

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1. Dewey, Logic, Theory of Inquiry, Introduction, New York 1939.
2. Ibid, p.10.

Practical success may be very important indication that the guiding principles and forms of thought were true. It cannot be the sole criterion of truth and logicality.

Pragmatism, (apart from Peirce, who is much more than a pragmatist) and all kinds of psychologism, can be also reproached for neglecting the linguistic aspect of thinking. It is impossible for logic to study directly human activity as such - neither the operations of thinking nor the practical operations of enquiry. Once it is understood that logical problems are only indirectly extra-linguistic problems, in so far as a language must be interpreted, it entails the necessity of paying much more attention to the syntactic and semantic problems than was ever realised by original pragmatists and psychologists. The same applies, of course, to any other classical conception of formal logic.

So far the result of our discussion seems to be:
1. We have found a common ground for discussion about logic in its linguistic aspect. In the present state of affairs every serious philosopher would have to agree that logic is <u>at least</u> a theory which investigates structures of meaningful symbols.
2. Asking what logic is beyond that, we have found the weakest existing criterion in modern conventionalism. We had to reject it as too weak because it fails to make certain assumptions explicit which are tacitly held by all logicians in so far

as their work is of any philosophical and scientific interest. As a consequence, no distinction can be made between logic and any game with symbols that satisfies the formal criteria of building artificial languages.

3. At the opposite extreme we have found realism which provides the strongest possible criterion identifying the conceptual structure of logic with the structure of reality. We have seen that this conception is too uncritical and that it completely fails to account for the extraordinary creativity in the field of logic in the last century.

4. An examination of the other doctrines between these two extremes shows us (i) that they are all one-sided, drawing out some important features of the body of knowledge which is generally considered as logic; and (ii) they are either close to realism in their dogmatism and attempt to establish a closed absolute system of forms, or they are close to nominalism in their exaggerated flexibility and relativity, which even comes to the point of allowing plurality of truth on the same question.

And so it is fairly clear that what we are looking for are such qualifications of the subject matter of logic as will make us conceive logic (a) as an objective science but not objective in any dogmatic, absolutist way; (b) as a highly flexible science but not flexible to the point of relativism and pluralism of truth.

What these vague predicates "objectivity" and "flexibility" mean is easiest to see when we apply them, at least in a very preliminary way, in our discourse about truth, which is the central notion of logic, and which we shall have to discuss later in much more detail.

Let us take for example a statement P which is scientifically established as true. To say that P is true, then, does not usually mean, unless we are pragmatists, that it is true for x but not for y. Provided (a) that y understands what P asserts, (b) that he has been acquainted with the reasons why we hold P to be true, (c) that he cannot give any reasons against the appraisal of P as true, except some very general philosophical doubts which can be directed more or less against all assertions, and finally (d) that y is a sane, normal person, who really wants to know the truth, we have the right to expect that y will agree that P is really true.

In such a way objectivity of truth here means that P is true independently of the appraisal of any particular individual or group.

By this qualification we have escaped the conventionalist's subjectivism, but the problem remains how to distinguish between "objective" in a relative sense and "objective" in the sense in which some metaphysical philosophers wish to use it, meaning by that term something eternal and unconditional, holding independently of human beings. Examples of these absolute truths are Bolzano's

"truths in themselves"[1] which belong to "propositions-in-themselves" (Sätze-an-sich). This way of speaking about truth has been particularly popular among the adherents of the phenomenological school. Husserl gave the famous example of the proposition which expresses the law of gravitation and which, in his opinion, would have been true even if neither Newton nor anyone else had ever discovered it. Of course, the thesis that there are truths-in-themselves is irrefutable in the sense that no one can ever show that they do not exist. In a similar way no one can ever show that ghosts, absolute spirits, immortal souls, etc. do not exist. If phenomenology claims that there is "an ideal sphere of pure essences" and if this sphere is neither located in the world of human experience and knowledge nor in the world of material objects we do not know where to look for it and we cannot either prove or disprove whatever we are told about it. Obviously the same applies to the opposite thesis. If someone emphatically denies that there are, or that there have ever been such imaginary entities as truth-in-themselves, propositions-in-themselves, ghosts, absolute spirits, immortal souls etc., his view is also irrefutable, because no amount of facts can ever prove that there is a single thing of the kind in question.

When put in this way, the problem is clearly of a metaphysical nature and cannot be solved, although the side which

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1. Bolzano, Science of Knowledge Sulzbach, 1837,S.17-45.

denies is in a much more comfortable position, the burden of proof lying always upon those who assert something.

This problem is in fact a problem of semantics. We cannot say anything definite about the existence of truth, but we can discuss what is the meaning of the predicate "true" and in which cases it is properly used. Then our problem can be put in this way: Which categories of expressions can be values of the range x in the semantic expression "True(x)" ?

From the point of view of a non-metaphysician, obviously only those symbols can be permitted as the values of the variable x which either refer to some at least indirectly observable facts (in the case of empitical truth) or which have a certain sense within the framework of a theory or system (in the case of logical truth). For example, the following two statements would be semantically correct:

1. "Jupiter has four satellites" is true.
2. "Five is greater than three" is true.

Of course in order to be semantically correct, i.e. meaningful, an assertion about the truth of an expression need not necessarily be true itself (true in a meta-meta-logical sense). What is here at issue is not the problem when our ascriptions of truth are true themselves, when we are right in saying that so-and-so is true. This is the problem of the <u>criteria of truth</u> of the ascriptions of the predicate "true" and we shall discuss it

latter. Here we are concerned only with one part of it: i.e. the problem of the criteria of meaning of the ascriptions of the predicate "true". In other words, our problem is: when does it make sense to say that so and so is true. And, as false sentences can also be meaningful, the non-metaphysician would agree that the following statements are also semantically correct although false in the ordinary interpretation of the words they contain.

3. "Mars is bigger than Jupiter" is true.

4. "Three is greater than five" is true.

Now the metaphysician would be much more liberal. He would allow as the values of the variable x in the above formula even the expressions which are neither related to experience nor to a certain logical theory as for example:

1. "The supreme being is the original ground of all things" is true.

2. "Pure essences (ideas, truths-in-themselves etc.) exist independently of human mind" is true.

3. "Eternal objects are related among themselves" is true.

The statements which are allowed here as the values of x in the formula "True (x)" are neither confirmable nor refutable and they cannot be shown to be either empirically or logically true. The meaning which an exponent of that view wishes to assign to the term "truth" is loose to the extent that it would be correct to speak even about religious and other mystical statements being true. The reason why we cannot follow him is that we do not

wish to allow ascriptions of truth in all these cases where we have no methods whatever to establish whether a certain statement is really true or false. To say that a statement is meaningful implies in our logic that there are at least in principle some methods of decision whether it is empirically or logically true. Therefore, we accept any proposition obtained from the scheme "x is true" by substituting a concrete expression for x as a meaningful proposition only if x can in principle be shown to be true by observation and verification or by being a valid formula in an interpreted formal system.

Of course, the metaphysician might argue that there are other means of establishing truth besides the procedures used in ordinary life, science and logic. He might refer to intuition, feeling, immediate impression of necessity, divine revelation, etc. Of course there is nothing to prevent him from doing so, only we wish to draw a demarcation line between these two meanings of the objectivity of truth.

In the <u>first</u> case objectivity of truth means independence of the approval of any particular individual or group. However, in order to be ascribed true a statement must be empirically verifiable or provable[1.] in a certain logical system. Objectivity is here relative to the methods and conditions in which the

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1. It should be understood that the class of "provable" statements comprises both axioms and provable theorems.

statement is established as true.

In the second case it is irrelevant for the objectivity of truth whether there are any methods for establishing the truth-value of a statement. In order to be accepted as true a statement need not be either empirically verifiable nor logically provable.

This is not the only difference between a relative and an absolute meaning of objectivity of truth. There are two ways of reacting when a situation arises in which we have to complete or revise or reject a statement which we believed to be true. The one way is to say that our belief was wrong but if the statement were true at all, it would have been absolutely true. Therefore all our empirical and logical statements are absolutely or eternally true if true at all. The second way is to say that truth is relative to a certain language, available evidence, theoretical reasons etc. so that whenever we change the context, truth gets a new sense and thus it is only within this transformed context that the old statement becomes false.

No matter which way of talking we choose there are some facts which both sides must take into account. Firstly a great number of our empirical statements refer to temporarily and spatially given events, which take place under some specifiable but not necessarily explicitly specified conditions. Our statements are necessarily always to some extent abstract in

the sense that no reference to all these conditions is made: therefore whenever we have a proposition which is normally regarded as true, it is possible to conceive a set of conditions in which it would not be true, e.g. it is not true that the orbits of the planets are elliptical unless we take the solar system as the spatial system to which we refer. The statement "Victoria is the queen of England" is no longer true unless we specify the interval of time during which Victoria reigned.

When we do so, our statement (and all similar statements which refer to particular events) is roughly speaking complete. However, what right do we have to say that it is absolutely complete and that even people who will live many thousand years later will not have to add some further specifications in order to make it more precise. E.G. "Victoria was the Queen of England in the year 1850" is true only under the conditions that the year of Christ's birth is taken as the starting point (and not the year of - let us say - Mohammed's escape from Mecca) and that the decimal system is taken as the number system. If we change the way of counting or the calendar, etc., the above sentance will be false. A good counter-argument would be, of course, that what was meant by the sentence still holds as true although it has to change its linguistic form. But then again "what is meant" is relative to those who "mean" it, i.e. some group of human beings.

Statements have their full meaning, and consequently can be regarded as true, only relatively to a large number of assumptions which are ordinarily taken for granted. Taken separately they are necessarily incomplete. This holds particularly for general statements.

Secondly, our empirical statements are not absolutely certain, in so far as we must never exclude the possibility of their revision in the light of some subsequently discovered facts, or of some new theory with a greater power of explanation, e.g. Newton's statements of the laws of mechanics were regarded as absolutely true for centuries. Now we know that they are not, although it would be wrong to say that they are false. In a limited range of phenomena (i.e. for the bodies of relatively great mass and small speed) the errors to which they lead us are so negligible that for all practical purposes they can still be taken as true.

Thirdly, as to the logical statements, it is quite clear nowadays that there is no absolute logical system and that the logical truth of a statement depends on the syntactic and semantic features of the logical system in which the statement is provable. In other words, logical truth is relative to the given system. The system itself is relative to certain criteria on the basis of which we decide which systems are to be regarded as logic. These criteria are relative to a given stage of the

development of science and philosophy, e.g. after the discovery of the logical paradoxes, some statements of the nineteenth century logic can no longer be considered as true.

In all cases of this sort a philosopher who does not wish to conceive truth as relative, would explain that we wrongly believed that a certain statement was true, which is, however, no reason to believe that the statements we are still holding are not absolutely true if they are true at all.

Although both ways of talking can account for all these facts, I find an absolutist's language in so far awkward as the term "truth" becomes quite empty. If, whenever we revise our statements, we must hold our past beliefs for absolutely false and always speak of the truth of our present beliefs in hypotheticals: "if p were true at all, it would be absolutely true"[1], then the term "truth" barely tells us anything. It seems simpler to speak of truth in a relative sense, i.e. relatively to a certain context of language, logic and available evidence. Then we may say: The truth of empirical statements is <u>relative</u> (a) to the set of conditions under which the event to which the statement refers takes place, (b) to the evidence available and to the theoretical considerations by which it can be supported. The

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1. To assert absolute truth categorically and still to allow the possibility of our being wrong seems to be a very odd attitude. "P is true absolutely" excludes "It might be the case that P is not true."

truth of logical statements is relative to a particular logic which is taken as a system of reference. The objectivity of truth has a definite meaning only if it is taken in a relative sense.

Now, how does all this apply to logic and in what sense can logic be conceived as an objective science?

Logic consists of a number of rules and principles and their constituents. All these are logical forms in so far as they represent the truth-conditions for all propositions of a given universe of discourse. We are at liberty to construct formal patterns of signs as we like and to interpret them as we like. However, only those formal calculi and semantical theories are philosophically interesting (can be accepted as an objective science) which besides satisfying purely formal criteria (of stating explicitly rules of formation, transformation and semantical rules) also fulfil the following conditions:

(a) whenever we can at least in principle substitute some[1] expressions with a definite meaning for indefinite

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1. This "some" means hereby a sub-class of all descriptive expressions. In fact, the semantics of a logical system must provide rules which state precisely (a) to which range of descriptive expression R the logical system S in question is applicable, or, in other words, which is the "universe of discourse" from which the descriptive expressions can be taken in order to replace the variables in S, and (b) which is the category of expressions of R to which a certain symbol in S is applicable. When the range of values for each variable in S is determined, it must then satisfy the condition that for all values of the variables in S we can transform the formulae of S into objectively true propositions.

expressions (variables) of a logical principle[1] (either axiom or theorem) we should get a concrete proposition which is objectively true in the sense of objectivity defined above.

(b) Whenever we use a rule of a system which purports to be a logic we should satisfy one of the conditions of the objective truth of a certain concrete proposition.[2] In the specific case of the rules of inference, we must get from O-true premises - O-true conclusions. However, this applies also to all other logical rules. A rule of definition, for instance, when used for the clarification of meaning of a certain term, is also one of the conditions for establishing the objective truth of a concrete proposition whose constituent the given term is.[3]

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1. Sometimes we might construct very trivial empirical propositions out of logical formulae. Sometimes logical principles will be applicable to empirical sciences only more or less indirectly, through the applicability of mathematical formulae which can be constructed out of them.

2. It will be seen later that these conditions consist of rules of meaning, rules of proof and rules of verification.

3. For instance, we cannot know whether the statement "The atom of hydrogen is composed of one proton and one electron" is objectively true unless the terms "atom", "hydrogen", "proton", "electron" etc. are defined, and for that purpose we need correct rules of definition. Of course not all arbitrarily chosen rules of definition are correct (not all of them are truth-conditions). If we are told that we should define all class-symbols in terms of some of their sub-classes, such a rule would constantly mislead us.

Expressing all this in an abbreviated way we can say that logic lays down the truth-conditions for a certain theory (or system of theories or just a collection of statements) and that logical forms are schemes for getting objectively true propositions.

It is to be noted that (a) here we always speak about true propositions with the given specifications, i.e. in a relative sense; (b) we do not commit ourselves to the assertions that these abstract schemes which we have called logical forms are identical with the structure of reality, whether it be with some sphere of pure norms and essences or with the "a priori" structure of reason or any other absolute structure of ontological entities.

Obviously, when the logical forms constantly fulfil their function of establishing O-truth conditions, we are bound to make some explanatory hypotheses and this issue will be discussed later. However it is quite sufficient for our present purpose just to examine the role which logical forms actually play in science and everyday life. This flexibility helps us to account for the fact that the same function of establishing O-truth conditions can be carried on by several overlapping and more or less different systems of logical forms. What is required from any of them is only that they furnish schemata which allow them to be transformed into true propositions for all values of their variables.

It is not difficult to see where the limit of flexibility in this conception of logic lies. If somebody constructs the syntax and semantics of the language of chess, or any other arbitrary syntax and semantics and claims that he has built up a new logic, what we shall have to do is: firstly, to see whether what he says is publicly comprehensible, and secondly, to check whether we really get objectively true propositions when we use the forms of his logic in the way described above.

Thus the conception of logic which we have been looking for, and which would provide logic with its objectivity without any sort of absolutism, and with its necessary flexibility without any sort of conventionalism and relativism, might be described in a vague way as follows:

"Logic is a science about the structure of symbols such that (a) they have an established public meaning, and (b) they provide general conditions of objective truth for all sentences of a given language."

Of course, this definition of logic is still far from being satisfactory unless such essential logical notions as "meaning" and "truth" which were used in it are more precisely defined.[1] Here we are at the real core of all the other problems. The concept of truth is obviously the central concept

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1. So far we have discussed only what objectivity of truth means and in what context the term "truth" can be used.

of logic which determines the whole interpretation of unified science; so much so that one might be right in saying quite shortly that logic is "the theory of truth".

However, the clarification of the concept of truth presupposes a clear account of meaning. Therefore our two next steps will be an analysis of these two problems.

Chapter II

The Public Meaning of the Symbols used in Logic

We have already said that nobody would be content to regard any arbitrary system of incomprehensible symbols as constituting a logic. This view has been generally accepted in the last two decades, so that it is now required that for a calculus to be regarded as a logic, it must be interpreted, i.e. supplemented by semantical rules which define the meaning of the symbols.

However there is no general agreement as to what the term "meaning" means. Here again we have a similar situation as in the problem of the nature of logic. We can classify some of the most important conceptions of "meaning" by putting them in order from the weakest to the strongest. The former means the minimum conditions and the simplest possible ontology presupposed, the latter involves one in the maximum ontological commitments. We shall be able to see that the first extreme, while having a great advantage in simplicity, fails in its explanatory function whereas the second, while not presenting so many difficulties, as an instrument of explanation, takes too much for granted in its ontology, "multiplying entities beyond necessity."

We shall, then, try to establish a conception of meaning which

1. presupposes the simplest possible ontology, and

2. succeeds in explaining the crucial facts in connection with the problem in question.

Our incomplete classification of the various theories of meaning might run as follows:

(1) The meaning of a sign is its relation to the other signs of a language. The language in question may be an artificial language. The basic terms are not given explicit meaning in any other language, they are simply introduced and defined in an implicit way by their use in the basic sentences: the meaning of all other terms of the given language is then defined in terms of these basic signs. This is the so-called syntactic meaning of a sign. The doctrine which adopts only this meaning of "meaning" is formalism (Hilbert, Carnap). "The only entities whose existence is taken for granted are "signs".

(2) The next theory of meaning in order of complexity can be broadly described as behaviourist, and it is held by many contemporary positivists [and pragmatists]. Here the attention is not restricted only to artificial languages with an unlimited freedom of postulation. The main interest is taken in the meaning of the words in ordinary language. The knowledge of the meaning of the

words is presupposed but the only criterion for that knowledge is found in linguistic behaviour. Therefore, the meaning of a word is defined as "its use in the language"[1]. (Wittgenstein) or as Ryle has put it, to say that x has some meaning is to say that x has some use, i.e. a "set of rules or conventions which regulate its use".[2] This theory presupposes not only the existence of signs but also the existence of people who use them. However the main point is the denial that there are any mental entities in human minds (concepts) which are expressed by linguistic sign . It is believed that they should be replaced by dispositions to use words in a certain way. Here, although dispositions are not conceived as acts or events, they are clearly extra-linguistic entities, which make the ontology of the behaviourist theory of meaning considerably more complicated than that of formalism.

A particular variant of behaviourism is the pragmatic theory of meaning. We shall note Charles Sanders Peirce's principle of pragmatism. "Consider what effects, that might conceivably have practical bearing, we conceive the objects of our conception to have. Then, our conception of these effects is

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1. Wittgenstein, Philosophical Investigations §43, p.20 Oxford 1945.
2. Weitz, Oxford Philosophy", The Philosophical Review 1953. No.12 pp.137-253.

the whole of our conception of the objects."[1] The reference to objects is not at all necessary in Peirce's theory of meaning as one can easily see from the other formulations.

What is common to all behaviourist theories, including Peirce's is the defining meaning entirely by our conduct, independently of any inner mental activity on the one hand and the relation to material objects on the other. However Peirce does not restrict criteria of meaning only to linguistic behaviour (use of words) but to the whole of our practical conduct. The minimum of his ontology comprises, therefore, signs and people who not only have dispositions (intentions, desires etc.) but also undertake practical actions which transform their surroundings.

b. Verifiability theory of meaning introduces the relation of signs to sensory experience. It requires that any meaningful statement should be capable of empirical verification. We find an early version of this theory again in Peirce as an (unnecessary) extension of his principle of pragmatism: "....A conception, that is the rational purport of a word or other expression, lies exclusively in its conceivable bearing upon the conduct of life:

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1. In order to ascertain "the meaning of an intellectual conception one should consider what practical consequences might conceivably result by necessity from the truth of that ~~assumption~~ conception, and the sum of these consequences will constitute the entire meaning of the conception." C.S. Peirce, "How to make Our Ideas Clear" Collected Papers Vol.V. p.9.

so that, since obviously nothing that might not result from experiment can have any direct bearing upon conduct, if one can define accurately all the experimental phenomena which the affirmation or denial of a concept could imply, one will have therein a complete definition of the concept, and there is absolutely nothing more to it.[1.]

This verifiability principle became later one of the fundamental theses of logical positivism. The meaning of a proposition was defined as the method of its verification. An amended version of this principle was proposed by Professor Ayer in a later edition of Language, Truth and Logic.[2.]

4. There is finally a large group of theories which explain the meaning as the relation between the sign and its designation i.e. that which a sign stands for. As this relation is commonly called semantic in modern logic, we shall consider all theories of this sort as semantic theories of meaning.

Of course there is a vast difference between them, depending on what sort of entities a philosopher is prepared to admit as the designata of the signs.

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1. C.S. Peirce, vol.V, p.412, 427.

2. A.J.Ayer, Language, Truth and Logic 1946, Introduction p.13. The reformulation of the principle has been discussed by O'Connor, Brown and Watling.

A. Some very extreme positivists refuse to commit themselves to the admission of any extralinguistic entities as designata. For them to say what a symbol means is not to relate it to an object but to explain (interpret) it in terms of other symbols. For the explanation (interpretation) of a higher-level language must be used. Here the meaning is entirely reduced to verbal definitions, i.e. it is again conceived as essentially the relation between signs themselves which was characteristic for the syntactic theory. The only difference seems to be in using two different languages (object-language and meta-language). However this is not necessary: the same language can be used for both object-language and meta-language on the condition that the "explicans" should be put into inverted commas. In such a way this conception of semantics is almost identical with the formalist treatment of the syntax, and perhaps one should call it pseudo-semantic.

B. A semantical theory of meaning based on empiricism introduces a distinction between "formal" and "descriptive" symbols. Whereas the meanings of the former may be given only by describing their relations to other symbols, the meaning of the latter is constituted by some relevant experiences, and may be defined in an ostensive way. As the "formal" symbols are to be defined in terms of descriptive ones, "no essential word in our vocabulary can have a meaning independent of experience"[1]. In view of this

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1. Russell, Inquiry into Meaning and Truth, p.365.

old empiricist belief Russell has formulated his principle of reducibility to acquaintance: We must attach some meaning to the words we use, if we are to speak significantly and not utter mere noise, and the meaning we attach to our words must be something with which we are acquainted."[1]

C. A semantic theory of meaning based on conceptualism admits abstract mental entities - concepts as designata of signs. However these concepts are not given a metaphysical status in a Platonic sense. They exist only in so far as they are given in the form of some basic mental functions or can be constructed by human mind.

The first view was held by Cassirer and other representatives of the new-Kantian school. Concepts were conceived as invariant "a priori" constituents of all human thought and experience.

From the point of view of another modern form of conceptualism to say that a sign has a meaning amounts to the assertion that it designates a concept which can be constructed in a finite number of operations. If this actual construction is impossible, the sign is meaningless although it might have been derived in accordance with some laws of classical logic (e.g. the law of excluded middle). This second view is held in modern logic by representatives of the intuitionist school (Brouwer, Weyl, Heyting, etc.).

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1. Russell, The Problems of Philosophy p.91.

4. Finally a realist semantic theory of meaning posits real objects and their properties and relations as the designata of the linguistic signs. In this sense Russell wrote in his early work, The Principles of Mathematics: "Words all have meaning, in the simple sense that they are symbols which stand for something other than themselves..... When 'a man' occurs in a proposition (e.g. 'I met a man in the street'), the proposition is not about the concept 'a man' but about something quite different, some actual biped denoted by the concept."[1]

Some realists have found it necessary to distinguish between "sense" and "reference" (Frege) or between "meaning" and "indication" (Russell). The first is essentially a thought, something that is expressed by a sign. The second is something which exists objectively and is indicated or denoted by the sign. To the same reference several senses can correspond, as Frege showed with the example of "the morning star" and "the evening star". These are two different senses, two different ways of presentation of the same object to which they refer. On the other hand, there are signs which have no reference at all, but definitely have a sense. "The words 'the celestial body most distant from the Earth' have a sense, but it is very doubtful if they also have a reference."[2]

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1. Russell, The Principles of Mathematics, London, 2nd ed., 1951, p.47

2. Translations from the Philosophical Writings of Gottlob Frege, ed. Geach and Black, Oxford 1952, p.58.

Frege made a further distinction between such references as we can speak about in a predicative way (concepts - references of predicates) and references of proper names (objects). It is characteristic of realists that they allow concepts to have an objective existence, independent of people and their thinking and language.

All these conceptions, although different, do not necessarily exclude each other because they emphasise the various aspects of the same problem of meaning. Therefore when we examine them we must pay attention not only to exaggerations and over-emphases in them, but also to the grains of truth which they contain.

So if we start with the formalist theory of meaning, it fails because the relation of a given sign to the other signs is not in itself enough in all cases and without any other conditions to show the meaning of the sign. We can be given a calculus with explicitly established relations between signs, but it might be the case that nobody understands anything of it. To convey to a person B any information about the meaning of a sign x, a person A must either use some language whose signs already have for both an established meaning, or he must relate some experience of B with the sign x. The second way obviously transcends the syntactic methods of defining meaning to which a formalist wants to confine himself. However, the first does not open better prospects. Here it is true the meaning of x is defined

exclusively in relation to some other signs, but then the problem is transferred to the meaning of those other signs and here we have the possibilities:

(a) either we shall get a "regressus ad infinitum" always presupposing some meaningful signs as granted, or,

(b) we shall use the words of the ordinary language for explicanda and then declare that here the definition of meaning is no longer necessary because everybody understands the words of the ordinary language. And this is perfectly true. Everybody does understand ordinary language. Only this is so because everybody was taught at least the basic words of the ordinary language in an ostensive way, by relating symbols to some repeatedly occurring experiences.

Thus conveying meaning through relating symbols to each other presupposes the existence of at least some symbols whose meaning is constituted by relating them to experiences. These basic symbols need not necessarily belong to ordinary language. The only condition which they must fulfil is that they must be defined in an extra-linguistic way.

This is why a syntactic explanation of meaning must be supplemented by other considerations. It is true, however, that at least one dimension of the meaning of the vast majority of signs can really be defined in relation to the other symbols.

Now we turn to the behaviourist theory. We should distinguish two variants of the theory,; linguistic behaviourism where the meaning of a symbol is determined as its use in the language, and pragmatic behaviourism where the whole of practical activity is taken as the criterion of meaning. As to the first variant, its criterion of meaning i.e. "the use of a symbol in a language" is, first of all, very vague. Whose use of a symbol is here in question? If we take a particular individual it might be possible to investigate his linguistic behaviour and to determine what he means by his words. However, if we want to observe the linguistic behaviour of a whole community, it would not only be practically a very difficult task but also, since we shall not always find general agreement in use, we should have to decide which is the group whose usage is to be taken as authoritative. The difficulty lies in the fact that it is not enough just to enumerate different uses and to conclude that there are correspondingly different meanings. Somehow we know that each use is not equally legitimate, and in some cases we speak also of abuses. For example, we say that some people abuse the words "democracy", "freedom", "peace", "humanism", etc. We do not say that they use these words in a different way which is as legitimate as ours - we want to say that they use them in an incorrect way. This implies that there is something like the limits of the "proper meaning" of a word, so that people who

deviate from it under some conditions use the word wrongly. There is nothing in behaviourism to account for such a claim. However, it is perfectly justifiable. When somebody begins to use a word in a queer way we do not want to say in all cases that something is wrong with his use. In many cases we shall say that he assigns a different meaning to his words and that he speaks a queer language. But in some cases we shall argue that his use is wrong, or that he abuses the terms, etc. There are a number of such examples in political propaganda. When Hitler used to talk about peace as the main aim of his policy, he did not want people to understand by peace something different from the ordinary meaning, e.g. war. The point is that what was necessary for his propaganda was just to let people understand the word "peace" in the usual sense and to let them believe that he was against war.

In a similar way, when a boy says to a girl "I love you" she is sometimes rather uncertain whether he really means it. And there is a whole class of cases where we ask ourselves "Well, he said so (let us assume that he always says so and that his use is linguistically perfectly consistent) but did he really mean it?" If the use of words is our only criterion we cannot go beyond it and ask whether the person who uses them is sincere.

From a commonsense point of view we make a distinction between the use of some verbal expression and its meaning.

We separate what the behaviourist wants to identify. And the fact is that linguistic behaviour can be at most a useful indication of the meaning of linguistic symbols. Even in that case it must be assumed that the person in question does not want to deceive us, in other words, that he really means what he says. And so we find the undefined meaning in the very assumption of our behaviourist investigation.

There is also another class of cases which cannot be accounted for on behaviourist principles. These are the unusual cases in which the use of certain words is not accompanied by what is usually called normal "consciousness" or awareness, e.g. the talk of lunatics, sleeping people, the quasi-talk of parrots, the use of linguistic signs by some machine, etc. If we do not hold that meaning is anything more than the use of verbal expressions it is impossible to see any difference relevant to the problem of meaning between these cases and the normal ones.

But, then, what is meaning besides linguistic behaviour? If we take again our two examples, Hitler talking about peace and our courting boy, we are immediately tempted to say: the best test of what they mean is the whole of their behaviour, their deeds, not only what they speak. Let us see what practical consequences follow from what they say and whether they will really act in that way. This would be of course the application of Peirce's <u>principle of pragmatism</u>. The strength of this principle is undoubted and in most cases the meaning of our

verbal expressions will be sufficiently shown through the whole of our practical conduct. For example, in Hitler's case speaking about peace is incompatible with attacking and invading small neutral countries, and we shall clearly decide that it was a deliberate abuse of the term "peace".

The second example, however, might present some difficulties. How shall we cope with the case of a man pretending to be in love, and acting with the purpose of seducing a girl? A good answer might be: wait for some time, and he will change his behaviour. But may be he will not change it for a long time. Or, if we take another example, may be he deceived himself and he sincerely believed for some time that he was in love, but then discovered that he was not. Where is the limit? How much of a man's behaviour is relevant to the meaning of his verbal symbols?

Peirce demanded that the sum of all practical consequences should be taken into account. "....The sum of these (practical) consequences will constitute the entire meaning of that conception".[1] However, very often we can neither make a complete list of the practical consequences nor do we need to. We cannot make them because they are inexhaustible. Peirce took a very simple case of the meaning of "lithium" as an example. He succeeded in

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[1] Peirce, Collected Works, vol V., p.9.

enumerating some twenty operations which should be undertaken by a chemist in order to get litium. However, although he chose the most favourable example, his list does not contain all practical operations. Chemistry has made enormous progress since his time, a number of isotopes of all chemical elements have been discovered and not a single genetic definition in terms of practical consequences would to-day be considered as exhaustive, precise and unambiguous.

On the other hand we need not compile these extremely long lists. The meaning of many abstract terms can be sufficiently explained by bringing them into relation to the other terms whose meaning is already presupposed although not necessarily explicitly given (the syntactic meaning). On the other hand, a certain number of crucial practical operations will suffice to posit the existence of objects which are designated by symbols. Then instead of infinite lists of practical consequences we shall refer to these more or less hypothetical objects and this will greatly simplify what we say about meaning. But, of course, this procedure transcends behaviourism[1] by including semantic considerations.

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1. It does not transcend Peirce's view on the whole. Although he laid much stress on practical consequences and in some formulations of his principle of pragmatism identified meaning with their sums, he did not deny the existence of objects and even presupposed them in his famous formulation of the principle in the article "How to Make Our Ideas Clear" Vol.V. p.2.

There is another even more essential difficulty with pragmatism. The term "practical consequences" has in itself various meanings depending on what is the aim of a certain practical activity. Let us ask, What is the meaning of "oil" in terms of practical consequences? We have sets of very different practical consequences depending on the various interests of various groups of people. One set will include such things as organising the exploitation of oil, trying to increase political influence in the country which possesses the oilsources, changing the governments, keeping certain military bases in order to defend the acquired positions. The other set will include such things as opening the struggle against foreigners who exploit the oil, corresponding diplomatic activity, negotiating, throwing bombs, etc. The third set will include operations which geologists have to undertake in order to find oil, etc.

Now the meaning of "oil" cannot be all this, not only because most of these practical actions are irrelevant to it, but also because they imply mutually incompatible meanings of the same word and most of them are purely arbitrary and devoid of any philosophical interest. What is philosophically interesting for the problem of meaning is not every practical action however oriented, but <u>those practical actions by which we can discover and test the objective truth of a verbal expression or of the verbal expressions whose constituent it is.</u>

to the conclusion is that a pragmatic analysis either leads to confusion or presupposes such concepts as "object" and "objective truth".

Now we come to the verifiability theory of meaning. This theory is not a theory of meaning in general. It covers only so-called "empirical" propositions. Within the framework of logical positivism it used to be conjoined with the syntactic conception of the meaning of logical statements.

The main difficulty with this theory is that it has not so far been possible to formulate it successfully. The previous current formulation of the principle of verifiability: "the meaning of a proposition is the method of its verification" suffered among other things from the consequence that it denied meaning to judgements of perception, that is to the propositions which are more certain than any others and which constitute the verification of all the other empirical propositions. This difficulty was met in Ayer's amended version of the verification principle[1] where a distinction between directly and indirectly verifiable statements was introduced, directly verifiable statements including observation-statements. However as O'Connor showed,[2] this formulation still had a few defects. His suggested

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1. Ayer, Language, Truth and Logic, 2nd ed., Introduction, p.13.
2. D.J. O'Connor, Some Consequences of Professor A.J. Ayer's Verification Principle, Analysis, January 1950, pp. 67-72.

solution aroused new objections by Watling and Brown[1]. and the formulation of the principle became more and more complicated.

Besides, it is not easy to see how to meet the objection that the principle of verification itself, not being an analytic proposition, must either be admitted as meaningless or must be verifiable itself, which is impossible.

Therefore it might be concluded that the empirical verifiability theory in its entirety presents a real contribution to the solution of the problem of meaning only on two conditions:

1. If it abandons the sharp distinction between analytic and synthetic (empirical) statements and allows at least three criteria of meaning (a) a syntactic criterion for analytic statements, (b) a verifiability criterion for empirical statements; and (c) some third criterion (pragmatic?) for such posulates as the principle of verification itself.

2. If it succeeds in providing a proper formulation of the principle of verifiability.

The second difficulty is only a part of the main difficulty of modern empiricism in its most thoroughgoing phenomenalist form: how are we to construct a language, whose terms would refer only to experiences and not to any external objects, such

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1. J.L. Watling and R. Brown – Amending the Verification Principle Analysis, March 1951, pp. 87-89.

that what ver we usually say about objects can be adequately expressed by talking about sense-data.

So far, starting with Carnap in Der Logische Aufbau der Welt, phenomenalists have not been successful in showing that this is possible. That is why in the last few years empiricists have no longer hesitated to speak about material things, avoiding only so far as possible admitting the existence of abstract entities as designata. But this is already a problem within the framework of the semantic theory of meaning.

In fact, no hard and fast line can be drawn between these various views. From thinking of meaning in terms of the internal relations between the symbols of a given language it is only a single step to introducing the person who brings the symbols into such relations (i.e. who uses them). A further step is the generalisation of linguistic behaviour to behaviour as a whole, i.e. practical conduct. Of course, it is impossible to separate practical activity from experience; that is why Peirce insisted so much that "every stimulus to action is derived from perception" and every purpose of action is to produce some sensible result".[1] The result is that particular attention has been paid just to those practical actions as being relevant to the problem of meaning, that represent some empirical verification or falsification of a

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1. C.S. Peirce, Collected Papers vol V. p.400.

given proposition. However, this conception of meaning as a method of verification had a serious defect in that it presupposed such statements as the utensils of the method to which the method itself could not apply without circularity. The authors of the verification principle have gradually changed the formulation of the principle in such a way that they have in fact made a transition from the rather pragmatic conception of meaning in terms of what-one-should-do-in-order-to-find-out-the-meaning-of-some-expression, to a semantic conception of meaning in terms of what-a-meaningful-expression-refers-to. So, for example, if we examine Ayer's amended version of the verification principle we shall see that it does not state what we should do in order to get a meaningful syntactic statement, at least it is not his primary concern, but rather what this meaningful syntactic statement is. Namely, it is either directly or indirectly verifiable; in any case it must be related to at least one observation-statement, i.e. in the final analysis to some experience. Therefore such a formulation of this principle represents already a transition to the empiricist semantic theory of meaning.

We have mentioned before that the validity of this theory depends on the possibility of constructing a satisfactory phenomenalist language, otherwise we should not be able to convey information about the entire meanings of our symbols. In other

words, it is presupposed that symbols express some extra-linguistic occurrences, but our language would be too poor to give a full description of them.

On the other hand, we have ordinary language with implicit assumptions which transcend a strictly empiricist (phenomenalist) theory of meaning. It is, namely, implicitly assumed in it that words designate objects and their properties and relations. Empirical philosophers[1] have claimed (a) that a phenomenalist language would not differ in any factual respect from a language with such naive-realist inclinations; (b) there would be only a linguistic difference, in favour of phenomenalist language, because the latter was much more precise: its symbols would refer to what we immediately and incorrigibly know, and not to any more or less obscure entities.

As to the first (a) a description in terms of experience would really be equivalent to a description in terms of material objects (although they would differ in their ontological import) only if the former were exhaustive enough as in the following simple case:

1. "x is round" means that the sense-datum expressed by the word "round" is the constituent of the conjunction of sense-data named by the symbol "x".

2. "x is round" means that some object has the property of being round.

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1. Ayer, Foundations of Empirical Knowledge, London, 1940

In this simple example it might be the case that what we know about the object x is nothing more than a certain conjunction of sense-data and what we know about the property "round" is nothing more than the sense-datum "round". Therefore we might agree that (1) and (2) give the same description expressed in two different philosophical languages.

However, in more complicated cases it will either be impossible to construct a description in the sense-data language, or this would not be equivalent to the material-object language. What we usually conceive as a material object differs from the corresponding sense-experience in at least the following two respects.

1. Rightly or wrongly we suppose that it exists in an external world independently of any experience.

2. Our conception of the object includes not only a conjunction of actual or possible observations, but also some conceptual constructions which are of a pragmatic nature and cannot be reduced to sensory experience. E.G. our conception of the sun includes as an element the temperature of 6000°C which no one ever experienced. This temperature means something to us because we have experience of the lower temperatures, and because the theoretical constructions and rules of operations by which we arrived at the concept of the temperature of the sun have at least a pragmatic meaning for us: we know that their use

will bring us to some experimentally verifiable results, but they themselves are not reducible to any actual or conceivable experience, as is the case with mathematical rules and concepts.

Even if we forget these two differences, and if we assume that material objects as designata are only suitable abbreviations of certain amount of experience, it does not seem possible to translate exhaustively and without any change of meaning a material-object language into a sense-datum language. The reason seems to be that what these abbreviations express are concentrations of an enormous and even, one might say, infinite number of actual and possible sense-data. Concepts of "material-objects" whose content is so purified that they do not contain anything more than conjunctions of sense-data, might be compared with logical concepts of <u>variables</u>, some of whose infinite number of values we must know if the concept is meaningful, but whose values we do not know exhaustively. Who could say that a simple expression like Rodin's "Thinker", which has a perfectly clear meaning when we know that there is a sculpture to which it refers, can be given an adequate meaning if this object (sculpture) were replaced by a conjunction of sense-data. We can never make a full list of them, particularly if we are supposed to include the past experiences of other people and possible experiences of future ones. To make another comparison using mathematical concepts: we know what an infinite set means, but we can never enumerate all its members.

The conclusion seems to be that relating signs to immediate experience gives us only a part of their meaning.

If we turn now to conceptualism we see that its ontology is much more liberal because the existence of concepts not only as habits, or rules of operating signs, but also as mental entities, constituents of the mind is assumed. However, their number is restricted only to those which can be constructed by a finite number of operations. (We ignore here the older, much more dogmatic, forms of conceptualism, associated sometimes with apriorism, as in Kant's philosophy, and refer only to the modern conceptualism represented by Brouwer's intuitionist's school.)

The validity of conceptualism depends mainly on the extent to which the supposed concepts can be accounted for in behaviourist terms, by explaining them as forms of using symbols, without the assumption of their existence as mental states or entities. If behaviourists succeeded in this reduction, the conceptualist thesis would become redundant which so far it has not.

Behaviourists prefer to call these conscious guiding forces dispositions, but one might rightly wonder (i) if the term "disposition" is to the slightest degree clearer than the term "concept"; and (ii) whether there is any difference between these two, provided that one does not separate thinking from language and concepts from the symbols by which they are

expressed. For, what behaviourists are concerned with is just not to allow this separation.[1] On the other hand, it is hard to see why one should insist on this separation. None of the functions which concepts perform and which make it necessary to introduce them into our ontology demands that they should have a separate existence, independently of the language in which they are expressed. These functions are:

1. regulating our use of symbols in accordance with our practical purpose in a given situation.

2. making communication with other people possible.

There is a sub-class of concepts which have some additional functions, namely

3. focussing the invariant features of our previous experience under certain conditions.

4. providing a criterion for selecting and classifying practically relevant experiences.

Suppose a class of experiences K (under conditions C) whose invariant characteristic was f. Once we have succeeded in abstracting f, given it a name (the symbol "F") and remembered the conditions C under which it can be identified, we have formed

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1. "And indeed I have no objection to saying that what I have described as 'talking to oneself' is, in some sense, a mental process. All that I have been concerned to show is that there are not two processes: a process of using certain words intelligently and a shadow process of 'thinking'" Ayer, Thinking and Meaning, London, 1947, p.3.

the concept F[1]. Now we have a rule for the use of the symbol "F" (we shall use it in order to refer to experiences obtainable under the conditions C.). We can communicate with the other people in so far as we succeed in making other people understand what invariant f we experience under C. Finally, we have now a criterion for the selection and classification of our future practically relevant experiences: we can decide which of them belong and which do not belong to the concept F.

These concepts which are directly concerned with experience (being formed on the basis of selecting its constant features under certain conditions) might be called concrete concepts. They are expressed by descriptive symbols. There are also abstract[2] concepts which in the best case, are only indirectly related to experience. Here belong higher-level scientific concepts (like quanta of energy), mathematical concepts (like numbers), logical and philosophical categories (like implication, truth, etc.). Concepts of this kind are often only rules for operating with certain symbols, as in the case of implication, imaginary numbers, etc.[3]

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1. Here deliberately we do not make the important distinction between private (subjective) and public (objective) concepts. The latter presuppose the concept of objective experience which will be introduced later.

2. One should bear in mind that the distinction "concrete-abstract" is relative, e.g. higher-level scientific concepts are abstract in relation to lower-level scientific concepts, but they are also concrete in relation to logical concepts.

3. In other cases they might be defined as classes of classes....of constant features of objective experience. (The meaning of the term "objective experience" will be explained later.)

Therefore, concepts in general (including both concrete and abstract) can be defined as sets of rules for using communicable symbols.[1] In special cases (of concrete concepts) these rules are established on the basis of abstracting some common features of experience under certain conditions.

Conceptualists tend to regard concepts as mental entities independent of language and of experience and this is the main source of their difficulties. They cannot give a satisfactory answer as to what sort of entities concepts are, in what way they exist in mind or elsewhere, how they have been created independently of experience, etc.

Of course while speaking about concepts one does not need to hypostatyse them. Particularly one does not need (nor is it possible) to separate them from the symbols by which they are expressed. A concept is separable from any particular symbol, and one can always invent a new symbol to express it. What is impossible is to have a concept without any linguistic expression which is correlated to it.

Another weakness of conceptualism is that it reduces meaning to concepts. It does not make sense for a conceptualist to speak about referring symbols to material objects, because

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1. A communicable symbol is a symbol which makes communication within a certain group of people possible.

these in the best case are beyond the limits of our knowledge (Kant's "things-in-themselves). Instead of objects we must always speak about concepts of objects. Objects are not something given in an external world, they are only the results of the synthetic construction of our reason (Cohen, Natrop, Cassirer).

This is a limitation of conceptualism which has been attacked by the realist theory of meaning. A realist claims that conceptualists do not tell us the full story of the meaning of a word by their reduction of the meaning to a relation between the symbol and a certain concept. When we think of the meanings of the word "table" or "Winston Churchill", he would say, we are not satisfied that the words refer only to some invariant mental occurrences called concepts. We mean by "table" a class of physical objects which exist independenly of us in an external world, and we can verify in a thousand ways that these objects are really there where we expect to find them. Or, we mean by "Winston Churchill" not only some conceptual description of properties but a living man who would exist in the spring of 1956 even if I had never been born. If I go to the place where he lives, I shall be able to see him, or get in various ways an enormous amount of evidence that he exists independently of our concept about him.

Similar considerations led realists to suggest that what in fact do constitute the meaning of symbols are real objects

and their properties and relations, concepts being themselves allowed into this class.

There is no problem for realists in answering any question about the meaning of some accepted symbol. The only thing they have to do is to assure us that there is a corresponding thing or event or property or relation or structure of relations, which is designated by the symbol and which constitutes its meaning.

The realists assume a threefold structure of the world: there is firstly, the structure of the symbols, secondly, the structure of concepts; and thirdly, the structure of reality. The naive realists identify the last two, therefore they find it extremely difficult to cope with concepts of imaginary entities like "golden mountain" etc. The more refined realists allow the possibility of concepts which do not correspond to anything in reality (Frege's "sense" without "reference"), or they distinguish between the "existence" of concrete things and the "subsistence" of abstract entities, or between the "actual" existence in the former and the "possible" existence in the latter case.[1.]

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1. See: Feibleman, "A Reply to Bertrand Russell's Introduction to the second edition of The Principles of Mathematics." The Philosophy of Bertrand Russell, The Library of Living Philosophers, vol.V, Evanston, Illinois, 1946.

The price which realism has to pay for such answers is that it is open to at least the following objections:

(1) Its ontology is completely uncritical. Any sceptic might enumerate a number of arguments against the existence of most of their "real" entities.

(2) Its whole doctrine is extremely inflexible, the consequence being that whenever we have to rearrange our conceptual schemes, the supposed structure of reality has to be readjusted. Now what sort of independent reality is it if it is bound to follow our behaviour?

(3) Realism does not account for the creativeness of thought, for the pragmatic character of at least some of our conceptual schemes which is responsible for there being a plurality of them which are applicable to the same kind of situation. What follows from it is mutual incompatibility between different theories operating in the same field, some corresponding to reality and being true, the others being false.

As (2) and (3) have already been discussed in Chapter I, I shall examine here only (1) in detail.

It is rather astonishing that many non-realist philosophers completely follow realists for half of the way, so far as particular objects are in question, but refuse to do so in relation to the meaning of abstract symbols. This attitude, adopted by many positivists after logical positivism ceased to

exist as a movement, is obviously in accordance with the old nominalist tradition. What was always at issue between nominalism and realism was only the existence of universals, not the existence of individual things.

The view which I am going to defend here is:

1. We cannot be <u>absolutely certain</u> either about the existence of particular things or the existence of abstract properties and relations;

2. We must <u>assume, in a hypothetical way, in accordance with certain criteria</u> not only the existence of particular things but also the existence of general properties and relations whenever these criteria are satisfied. The difference does not amount therefore to a cleavage; it is only relative, in the sense that the reasons for the former give a greater likelihood of existence than the reasons for the latter.

As to the first, the lack of absolute certainty here means the impossibility of proof. No logical argument can <u>prove</u> the existence of anything; the way in which all logical reasoning proceeds is: if so-and-so, then so-and-so, whereby nothing about the existence of any so-and-so is entailed. Further, no experience can ever give <u>sufficient evidence</u> for the existence of anything. If there are such things as optical and acoustic illusions, hallucinations, visions, etc. then the probability that we are not deceived in a particular case will never be greater than $\frac{n-1}{n}$, no matter how great n is.

But all this does not in the least imply that we can never have theoretically and practically every good reason to assume that not only our perceptions and concepts, but also the corresponding material objects and their properties and relations do exist. All the sceptical reasons for doubt demand only a little more flexibility than a realist is prepared to accept. This flexibility can be attained by satisfying two essential requirements.

1. Instead of saying that we are absolutely sure that there are such and such material objects, we shall be wiser to claim only that under certain criteria we are justified in assuming the existence of certain objects and their properties and relations. This difference in saying reflects the difference in having all the reasons one might have and having sufficiently good reasons, or the difference between absolute certainty and more or less high probability.

2. Instead of saying that our knowledge of material objects represents a full description of them and that, therefore, whatever we predicate of the terms by which they are designated corresponds to their real characteristics, and there is nothing more and nothing less in them, we shall be rather more careful and allow an essential discrepancy between our knowledge of objects, and objects themselves, provided that they really exist. In such a way, in opposition to realistic dogmatism all our

ontology becomes of a hypothetical and rather flexible nature, open to revision whenever it fails to conform to our adopted criteria of objectivity.

What are these criteria for assuming the existence of an object?

(1) There must be a concept F, i.e. in the simplest cases[1] knowledge of some constant characteristics of a class of experiences, which provides a set of rules for the use of some symbol "F".

(2) This concept must enable us directly or indirectly to predict some future experiences. What is here very important is that the rules for prediction implied by the concept must be invariant under all transformations of different conditions under which various particular observers might wish to check them. In other words, what is implied by the concept must be publicly verifiable, independently of any individual observer and his particular conditions.

(3) Practical actions undertaken in order to get the predicted experiences must be repeatedly and intersubjectively successful.

When these three conditions have been satisfied we are allowed to conclude: there are good reasons to suppose the existence of some objective F_o which is described in an approximate

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1. In the more complicated cases the connection between concept and experience might be rather remote and indirect.

way by our concept F_c. Here we call this relation between F_c and F_o an approximate (or relatively adequate) description in order to leave space for later readjustments of F_c in the light of subsequent investigations.

Now the essential point is: what is there to justify our conclusion from the fact that the conditions (1)-(3) of our criteria have been fulfilled, the conclusion, namely, that "there are good reasons to suppose the existence of some objective F_o".

What justifies this conclusion is that it is by far the most natural and convincing explanation of the situation described by the premises. For, what the premises describe in a very simplified way is this:

1. Whenever a set of conditions C was given, some F appeared to us.
2. We make a hypothesis that C will also be conjoined with F in the future.
3. Then we act and create C ourselves. In all cases, again and again and not only I, but also all the other people interested in the matter, observe that the occurrence of F follows.

What is here the <u>crucial</u> point is that if we exclude the possibility of the existence of some objective F, which appears to us, the probability of the coincidence of our prediction, practical actions and occurrences of F becomes

infinitely small because there were too many other possibilities.

The theory which best explains this coincidence is that there is a constant connection (a functional relationship) between C and F and that both are objective states of affairs, which accounts for the given regularity in our and other people's experience.

When such a flexible view of the nature of the existence of "real objects" is once adopted, it is hard to see what the essential (qualitative) difference is in the ontological status of particular objects and their general properties and relations. There is a difference, but it is a difference of degree, depending on whether the conditions of our criterion of objectivity (1) - (3) can be applied in a satisfactory way.

Concepts like "tables", "chairs" and "houses" are (i) directly related to experience: it is rather obvious that they are descriptions of some constant features of our experience, (ii) This makes it very easy to see what future experiences one might have if our concepts are adequate[1]. (iii) There is no problem about what practical actions one should undertake in order to check these predictions. Besides, an enormous number of people do this checking every day and every hour in every possible way. Therefore we can be completely satisfied that the

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1. For example, if our concept of pencil is adequate, we shall see written marks when we make appropriate movements with any particular pencil on a sheet of paper.

conditions of our criteria are fulfilled and that in relation to the objects of this kind there is less reason for scepticism than to anything else.

On the other hand, when we have to do with such theoretical constructions as "curved space" and "quanta of energy" we are much more careful. These concepts have been formed quite often as the result of very abstract reasoning regardless of any experience, or as pragmatic devices constructed in order to solve some theoretical difficulty without any idea whether they will work or not. It is not always very clear what bearing on our experience they have, nor what practical actions one should undertake in order to establish whether they are adequate. Even when we have reached some verification, as in the case of the concept "the curvature of space", by measuring the aberration of the light rays near great masses, we are not satisfied by one single verification, because there might be some other explanation of the observed effect.

Therefore, there is no one-to-one correspondence between concepts and the assumed objective features of reality. There are concepts such that we hesitate to enter into any ontological commitments, there are also concepts to which we emphatically deny any objective correlate (fairies, witches, centaurs, etc.).

However, such a state of affairs does not justify our drawing a sharp demarcation line between concepts of particulars whose designata are objective things and persons, and concepts

of general qualities and relations which - we are told - do not refer to anything existent. Where the demarcation line should be drawn depends on the whole of knowledge in the given field. We can be much more certain of the existence of quanta of energy or of gravitation or of heredity etc. than of the existence of, let us say, the Greek philosopher Leucippus.

In conclusion, we might classify the concepts and the symbols by which they are expressed into the four following groups:

(1) Adequate referential concepts. Whenever there is a concrete concept which satisfies the conditions (1)-(3) of our criterion of objectivity, we may assume, with more or less uncertainty (which is for all practical purposes negligible) that there is an object (or property or relation) which exists independently of any particular observer.

Here a further subdivision can be made in so far as we distinguish between those adequate referential concepts which denote ~~one~~ individual objects (or which are "instantiated") and those which denote classes, properties and relations. Nominalists would wish to exclude this second sub-class from the class of referential concepts. The meaning that they give to the word "reference" is rather narrow in so far as it is assumed that symbols may refer only to individual things. Here the term "reference" is used in a wider sense, covering also

those cases where we have good reason to think that a symbol denotes a property or a relation of a class of objects which really exist.

(ii) Inadequate referential concepts. Here the symbols which express these concepts refer to some hypothetical objects, but we have no reason to believe that these objects really exist. This is the case with mythological symbols (like centaur), some higher-level scientific concepts which failed to explain relevant phenomena (e.g. phlogiston, ether) etc.

(iii) Applicable non-referential concepts. Here symbols do not refer to objects or properties and relations of objects. Examples of these concepts are abstract mathematical and logical concepts such as number, differential, universal operator, truth, etc. They are not directly connected with experience, and it is not possible to say what "invariant features of past experience" they express. However, they have a certain bearing on our experience in so far as they provide rules for organising referential concepts and operating with them. They are applicable in so far as these rules lead to practically verifiable results.

(iv) Inapplicable referential concepts. Here belong those abstract concepts which fail in their operational functions, i.e. when applied they do not lead us to verifiable propositions. Such is, for example, Leibniz's concept of "logical subtraction" or the paradoxical concept of the "class which contains itself as an element", etc.

We hav got this classification by combining two principles of division: (a) whether a symbol does or does not refer to a certain objective state of affairs (whether it is or is not a descriptive symbol); (b) whether the concept which is expressed by the symbol does or does not enable us (at least indirectly) to obtain new knowledge and make correct predictions (whether it is successful or fail).

As to (a) the main difference between the groups (i) and (ii) on the one hand and (iii) and (iv) on the other might be expressed in Frege's and Quine's terminology by saying that whereas in the meaning of the symbols which belong to the first two groups one might distinguish between "sense" and "reference", the symbols which belong to the second two groups have only their "sense".

As to (b) one might say in anticipation that adequate and applicable concepts are constituents of empirically or logically true propositions, whereas the presence of an inadequate or inapplicable concept makes a proposition false, or at least indeterminate regarding its truth value.

Now we can try to answer the question: What is the meaning of a symbol x? A symbol x has a meaning if it has sense or both sense and reference.

The sense of x may be explicitly stated by giving a verbal definition in terms of a set of communicable symbols, but being

explicitly defined is not a necessary condition for x to be meaningful. The implicit sense of x is a concept[1] which can be applied to a class of cases. The concept in question should be regarded here as the intension of the meaning of x, and the range of cases to which the concept is applicable constitutes the extension of the meaning of x.

The reference of x again can be explicitly described by a verbal definition; however, this explication is not a necessary condition for x to be a referential symbol.

The implicit reference of x is an individual object (if x is a proper name) a class of objects (if x is a class-symbol) a property (if x is a one-termed predicate i.e. attribute) a relation (if x is a many-termed predicate) or a fact (if x is a sentence).

It is assumed that objects and classes of objects are characterised by certain properties and relations and, conversely, properties and relations do not exist except as characteristic features of objects and classes of objects. The latter objects and their classes are extensions of reference and the former are intensions of reference.

Now we should pass to the problem of objective or public meaning. We must clearly distinguish between private and public meaning because the conditions that a system of symbols must satisfy in order to be accepted as a logic comprise the

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1. In the case that x is a term; if x is a sentence, then its sense is a proposition.

latter, not the former. If we cannot interpret a set of symbols then we cannot regard them as a system of logic. If someone constructs a system in a queer symbolic language and dies without leaving any key for translation into ordinary language (or any language whose symbols have a public meaning) it does not make sense to speak about his system as a logic. It is irrelevant here what meanings were assigned to the symbols by the author. It is also irrelevant whether one day someone might translate the manuscript. So long as this has not been done, we have only strings of marks on paper. Here the situation is similar to that where someone constructs a new system in his mind but never formulates it. Knowing that he was a great philosopher and that he talked to his friends about his work on a new logic (never giving any particulars about it), we might hope that one day we may discover a manuscript. However, until that time his secret unformulated thoughts mean as much as the formulated but hopelessly incomprehensible thoughts in our first case.

Therefore, in order for a system to be acknowledged as logical, its symbols must first of all have an <u>objective, public meaning</u>, i.e. <u>a meaning which is invariant under all transformations of subjective conditions, experiences, etc., and which is therefore communicable</u>.

The easiest way to make sure that a language has a public meaning and that it is communicable, is to translate it

into ordinary language, because words of ordinary language already have a meaning which is, roughly speaking,[1] invariant not only under transformations of subjective conditions, experiences, etc., but also under transformations of particular forms of ordinary language in various countries. The ordinary language is the product of a long history; the meanings of its words are the generalised and crystallised experiences of an enormous number of human generations.

So we have come to the concept of "objective experience" which is of fundamental importance for building up a definition of public meaning and of objective truth. That there are some invariant features of the experiences of various people of a certain community, under some given conditions c, in some temporal interval t, we come to believe by observing human behaviour. The fact is, that under given specifications, we can communicate with other people.

We use certain symbols: words, gestures, facial expressions, etc. and other people react to them in the same way as we expect them to, and in the same way as we do when they use the corresponding symbols. This use of symbols enables people to cooperate, and this is one of the fundamental facts of human life in society.

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1. Only "roughly speaking" because otherwise in a strict sense this invariancy often does not take place, particularly in politics, literary criticism, etc.

Now in order to explain this fact we postulate that there are some constant features of human experience (emotions, etc.) relative to a given system of references (to a particular community, given conditions, spatio-temporal continuum, etc.). Of course, there are excellent possibilities for a sceptic to attack the notion of objective experience. He might rightly object that each individual knows only his own sensations, feelings, etc. and that one can never enter into another's mind in order to see what happens there, and to make comparisons. From this he would wrongly conclude that, therefore, it is impossible in principle to establish which elements are constant in the given context. One of the tacit premises in this argument would be that things must be directly observable if we want to speak about their constant features. No doubt, this assumption is false. If from the observation of the configuration of small drops of water in Wilson's cloud chamber we can infer something about the general features of electrons, neutrons, etc., we may also on the basis of the observation of the facts of human communication and cooperation build up explanatory hypotheses about the general features of other people's experiences. In both cases, whether the hypothesis will be confirmed or refuted depends on the reports of the other people's observation. If they all agree with the hypothesis, we say that it works.

It has been pointed out that the essential difference between two cases consists in the fact that whereas material objects are, at least in principle, directly observable, other people's experiences can never be directly checked for reasons of principle. Even if we accept this argument[1] it would still be irrelevant. We do not believe in the truth of our statements about electrons because we hope that some day we might be able to observe them directly, but because we are satisfied with the evidence available through indirect observations. A similar kind of evidence obtained through observation of people's behaviour is at our disposal for making general hypotheses about the mental states of groups of people. This is not the only kind of factual material which is available. We can also use the life-histories of the individuals concerned, their introspective reports, the data about the habits and traditions of the given community, etc.

Relying on facts of this sort and on empirical generalisations obtained by induction, one might construct a special logic[2] for social psychology in general, and for the social psychology of a certain society in particular. In such a way we might get statements about constant features of experience which are both inductively and deductively supported.

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1. We need not necessarily accept it. There are micro-physical phenomena which are in principle observable only indirectly (the principle of Heisenberg). This applies also to many astronomic phenomena which occur either too far in space, or under conditions unbearable for human life, etc. What to say, then, of historical, geological events, etc.?
2. The problem of special logic will be discussed later.

Therefore, it is legitimate to speak about "objective experience" in the sense of constant features of experience of various people of a certain group under some specifiable circumstances.

Without this (or some corresponding) concept, it is hard to explain how a language can be publicly comprehensible. We can learn the fundamental symbols of any artificial language by defining them in words of a natural language, but then we cannot know the meanings of the words of the natural language unless we have learned at least a small part of them by correlating our individual experiences with the public experience which our concepts are based on. So symbols play here only the role of mediators. If the word "ball" expresses an invariant (objective) experience of something spherical, made of elastic material, etc., a child learns it only when he correlates his own observations of something spherical, elastic, capable of bouncing when thrown against a hard surface, with the pronounciation of the word "ball". This word is a link which connects his experiences with the experiences of other people. Once the child catches the first meanings of the words - usually quite concrete, referential - he is sufficiently equipped to proceed by being taught the meanings of other words in terms of these first ones, defined in an ostensive way.

It is not necessary that this function of ordinary language as a universal instrument of communicability should be

played by any existing natural language. What is here named "ordinary" language is a language whose essential characteristic is that at least a part of its expressions (descriptive or "referential" in our terminology) is directly related to experience and can be learnt only in an ostensive way, the other part of its expressions being definable in terms of these "descriptive" expressions and a number of connectives whose use it is possible to grasp. People may agree by convention to teach their children "esperanto" or any other artificial language. In such a way "esperanto" might become the ordinary language one day.

Therefore, this meaning of "ordinary" language being accepted, we can avoid any explicit mention of the terms "objective experience", "public meaning", "communicability" etc., and express the same idea in a simple way: <u>In order to be acknowledged as a logical system, a system of symbols must satisfy the condition of being directly or indirectly translatable into ordinary language.</u>

"Directly or indirectly" here means that between the given language L_1 and the ordinary language L_o a sequence of other languages can be interpolated, such that the symbols of L_1 are given meaning in terms of L_2, symbols of L_2 in term are defined in terms of L_3 etc., the last member of the sequence L_n being translatable into L_o.

The expressions, "definable" or "translatable" in terms of L_o are synonymous with "having meaning in terms of L_o" and with "having a public meaning".

Therefore, we can also define now what is the interpretation of a formal calculus in general, and logical interpretation in particular:

If the formal calculus in question is a system of symbols S_1, then its interpretation is the system S_2 which satisfies the following conditions:

1. The symbols of S_1 are given meaning in terms of S_2.
2. The system S_2 contains all the sentences of S_1 in the translated form.
3. The system S_2 contains the same rules of transformation in the translated form.
4. All sentences which are formally valid in S_1 are also formally valid in S_2.

The logical interpretation of S_1 must satisfy conditions 1 and 4, with the following specifications:

(i) The symbols of S_1 must be given meaning directly or indirectly in terms of ordinary language, which means that either S_2 is an ordinary language or it is a member of a sequence of languages whose last member can be translated into ordinary language.

(ii) All sentences which are formally valid in S_1 must be logically true in S_2.

These two conditions are in fact the same as those that we have taken as the constituents of our concept of logic.

What we are still lacking here is the definition of truth, and this is the problem to which the next chapter will be devoted. The result of the long discussion in this chapter is that we have got not only the conception of meaning but also most of the other extra-linguistic elements necessary for the definition of truth. The question is only whether this conception of meaning really satisfies the criterion adopted at the beginning:

1. to presuppose the simplest possible ontology.
2. to explain all the crucial facts of the problem of meaning.

If we enumerate now these crucial facts we shall get a list like this:

(a) We cannot express the meaning of a symbol unless we bring it into a set of relations with the other symbols.

(b) This "bringing into a set of relations with the other symbols" can occur either through its definition or through its use, the first being the explicit and the second the implicit way of expressing the meaning of x.

(c) A definition can be understood if and only if its constituents are terms directly or indirectly related to experience (through their translatability into ordinary language).

(d) The use of symbols can be irregular, incoherent, automatic, purposeless, inconsistent with the rest of the user's

behaviour. In all these cases it is obvious that either the user does not know what he means by his symbols or he uses them insincerely, deliberately intending to deceive us.

(e) On the other hand, when the use of symbols is regular, coherent, purposive, we hold that the user knows their meaning, and if his use of them is consistent with the rest of his behaviour we say that his use of symbols is in accordance with what he means by them.

(f) In the simplest cases of expressions like "tables" and "chairs" this knowledge of meaning consists in knowing what actual situation would be the proper occasion for its use, or, what amounts to the same thing, what are the permanent features of experience under given conditions which are expressed by the symbol.

(g) In the more complicated cases of abstract symbols like "number","the state" etc., the knowledge of meaning consists in knowing either how such an abstract symbol can be defined in terms of concrete ones, or in knowing how it can be used as a rule for manipulating the more concrete terms.

(h) Before a man is able to create his own artificial language he is taught some basic concrete expressions of the ordinary language in an ostensive way by correlating with them his own experiences under given conditions. In this process of

learning by acquaintance, his responses under given conditions are being corrected by other people, until it has been established that what they all correlate with the use of a certain symbol under certain conditions is a similar experience.

(i) When this one-to-many correlation between some invariant features of experience of various people has been established, a person is able to communicate with others. This entails that the symbols they use have roughly the same meaning (public meaning).

(j) Knowledge of the public meaning of a symbol enables one to make predictions concerning future experiences and to verify them by creating conditions in which the experiences can normally be expected.

(k) There is an extremely small probability that the permanent coincidence between predicted and actual experiences is a matter of chance.

Now what minimum of ontological entities did we have to assume in order to account for these facts?

1. Symbols,
2. People who use the symbols and who perform various other practical operations of an extralinguistic nature.
3. Sensory experience.

4. Concepts and propositions which do not exist separately from their linguistic counterpart - terms and sentences - but cannot be reduced to them since they are rules for their consistent use.

5. States of affairs (objects, events, etc.) which exist in the same hypothetical but highly probable sense in which exist (although with different degrees of uncertainty):

   (a) my body

   (b) symbols I have never seen or heard

   (c) other people with their sensory experiences, concepts, practical actions, etc.

The test of existence in all these cases is:

   (a) the coherence of my concept corresponding to any of these entities with the rest of my knowledge.

   (b) practical verifiability i.e. (i) possibility of derivation from such a concept consequences concerning future experiences under certain conditions; (ii) creating these conditions; (iii) having the expected experiences.

The ontology obtained in such a way is obviously much simpler and much more critical than the realist one. The difference is not only in the number of entities, because our criterion of objectivity will exclude a number of quasi-objects[1].

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1. If <u>all</u> our concepts are identical or correspond to some objective state of affairs, then one would be obliged to accept that there are objects which are correlated with such concepts as implication, irrational number, square root etc. These should be considered as quasi-objects.

from our world, but also in the much more important fact that we do not commit ourselves to asserting that we are absolutely certain about what objects and facts are. We assume that they exist and that there is a certain correlation between them and our concepts. However we make allowance for a certain discrepancy between them, which explains why sometimes our concepts are only partially applicable or even misleading.

Pap wrote, for example, "Now the semantic conception of truth amounts to this: '(p) is true' is synonymous with the simple assertion (p); and correspondingly '(p) is false' is synonymous with the simple assertion of the denial of (p)."[1]

"If you say 'that snow is white corresponds to reality' you seem to have two related terms, the proposition and reality. But the quoted sentence is synonymous with 'it is the case that snow is white', and the latter sentence is synonymous with 'snow is white'. Analysis shows, therefore, that in using the phrase 'it is the case that . . .' we add no information about the proposition asserted, but simply assert the proposition in an emphatic way."[2]

To such an interpretation of Tarski's formula it should be objected firstly, that it deviates considerably from the "intentions" of the correspondence theory of truth and consequently of Tarski himself, and secondly, that it is rather trivial and philosophically uninteresting.

As to the first, Pap is wrong, I think, in holding that the sentences "that snow is white corresponds to reality" and "snow is white" are synonymous. The realists may be reproached for giving formally imprecise definitions, but what they mean by reality of some x is certainly not the same as someone's emphatic assertion of x. Reality of x implies that x exists independently of any observer and that, if it is observable at all, it is publicly

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1. Arthur Pap, Elements of Analytic Philosophy, New York, 1949, p.350.

2. Ibid.pp.355-356.

observable. Nothing of the kind is implied by someone's assertion of x; no matter how emphatic it were, it might be purely subjective. E.g. one might assert that there is a devil in the room. ~~The meaning of~~ His assertion does not necessarily entail that anyone else should see the devil - it might express only his own personal conviction. Now if he adds that it is a fact that the devil is in the room the meaning of this assertion, no matter whether it is right or wrong, is stronger in so far as it entails the possibility of a public test. Of course one can alter the meaning of the terms "reality" and "fact", and this is exactly what Pap does. The point is that in that case the correspondence theory is misinterpreted.

We come to the same conclsuion when we compare the meaning which Tarski assigned to his general scheme of the definition of truth with Pap's interpretation. Tarski made it clear that his proposed scheme is only a precise symbolic translation of the following definition expressed in an informal language: " a true statement is a statement which asserts that things are so-and-so and things are indeed so-and-so "[1] Although this definition is"not formally quite clear" Tarski holds that its "intuitive meaning and general intention are clear and understandable". In his opinion, the task of a semantic definition "is to make this intention more precise and to give it a correct form".[2]

1."Eine wahre Aussage ist eine Aussage, welche besagt dass die Sachen sich so und so verhallen . . . und die Sachen verhalten sich eben so und so." Op.cit. ~~para~~S. 268.

2. op.cit. ~~para~~S. 268.

~~Although this definition is not formally quite clear~~

So Tarski leaves us in no doubt that "p" in his scheme refers to the actual behaviour of things (at least in so far as p is an empirical proposition). Now in Paps interpretation "p" is only an "emphatic reassertion", which means much less because it entails no reference to extra-linguistic entities.

Such an interpretation is possible and quite legitimate,[1] only it is trivial and philosophically uninteresting, and this is our second objection. If the only condition which "p" in Tarski's formula should satisfy is to be an emphatic reassertion, then this formula becomes so weak that it tell us almost nothing about the meaning of truth. "x is true if and only if p." Now I assert p, i.e. I claim the proposition p to be true, therefore the sentence x is true. This is simply begging the question. What we are interested in is what it means to assert p. (so that it entails the truth of x) The realist answer is: to assert p means that p refers to some fact. This answer is already philosophically interesting, only it needs further explanation. What is to count as a fact? Or, to put it in another way, What conditions ought p to satisfy in order to belong to ~~THE CLASS OF PROPOSITIONS WHICH REFER~~ (adequately) to facts?[2] And consequently, What

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1. It is legitimate because the formula has a hypothetical form. The proposition "'The earth is flat' is true if and only if the earth is flat" is valid but trivial because this formula is satisfiable by all assertions.

2. As we are concerned here with the problem of the meaning of truth, not with the problem of criteria of actual truth, to mean "a referential proposition" should not be confused with to be an adequate referential proposition.

conditions should p satisfy in order to entail the truth ofits name "x"?

I think that determining these conditions is the best way of constructing the defintion of truth, on the basis of Tarski's general scheme.

Two other possibilities are these: (i) to specify a privileged group of people whose assertions of p entail the truth of x; (ii) to posit facts as the undefined terms of the language.

As to (i) if we like to keep the language of "assertions" and "reassertions in an emphatic way" we might make the following qualifications: p must be asserted by the community of experts or specialists in the given field. Here first of all we cannot avoid taking a temporal parameter into account. It must be understood either that the evaluation of x is made at the same time as p is asserted by the community of experts, or that experts would still agree in asserting p at the time when x's truth value is decided. Otherwise it might happen that the truth of x is claimed on the basis of some obsolete and wrong assertion of p in the past.

Now this qualification, although sufficient in most cases,does not work in some of them. Whenever there is a revolution in a science, such as that caused by Darwin's theory in biology, Einstein's theory in physics, etc., we have clashes among experts themselves, and what repeatedly happens is that the minority,or a single man, is right, whereas the vast majority are wrong. Therefore we should conclude that conditioning truth by the assertibility by the experts does not avoid too much upon counting heads,"① and the best thing would be to regard this procedure as merely approximate.

① Another objection to this view was made by professor Ayer in his "Verification and Experience" (Proceedings of the Aristotelian Society, 1937). It consists in pointing out that when we want to answer the question how it is established that the experts agree we get a vicious regress.

We may proceed further by examining what the advantages were of the attitude of these revolutionaries in science in relation to the conservative majority which opposed them. Obviously, owing to these advantages their views became generally accepted by specialists at some later time. We can imagine that these are roughly the same advantages as those which make the judgements of these specialists more reliable than those of ordinary laymen.

However, at this point we have already abandoned the method of defining truth through assertibility by some privileged group of people, and accepted a method of establishing conditions which a proposition should satisfy in order to belong to adequate referential propositions (the advantages mentioned above are exactly these conditions).

(ii) The second possible method consists in positing facts as undefined terms of the language. Tarksi's procedure belongs to this group. The terms of the meta- language in which he attempted the definition of truth include at least one term with an ontological meaning, namely "Gegenstand" (object).

And his procedure can be defended by the following arguments: (a) In any system of definitions some terms must be presupposed. Here if one wants to construct a definition of truth, general enough to include factual truth as a specific case, one must presuppose terms like "facts", or "objects" or "states of affairs", etc. (b) Although these posited terms are not explicitly defined, they receive their implicit definitions through their use, through their interconnections with the other terms of the system.

Functioning as a "definiens" in an explicit sense entails functioning at the same time and in the same context as the "definiendum" in an implicit sense. By relating various concepts to facts we gradually come to grasp what facts are, although no definition is given.

Methodologically this attitude is quite legitimate at the level of semantics, if semantics is taken in its ordinary meaning of semantics of logic, ie. as the meta-theory of a logical formal calculus. At that level only definitions of logical terms must be given, whereas extra-logical terms such as "objects", "properties" and "facts" may be left undefined. However at the next higher level of the general theory of logic (being a part of epistemology) terms like "objects", "facts", "referential concepts", "referential propositions", etc. should at least be explained in an informal way. Here "informal" means "without presupposing any explicitly stated logical system as something given, and consequently without using a "strictly deductive procedure".

As our discussion of truth is on the level of the general theory of logic and not on that of ordinary semantics, we should obviously accept this procedure of more or less informal explanation.

Our problem is then: which propositions are usually taken to count as propositions which adequately refer to facts? The best approach to this problem would be, I believe, by examining the history of science. Propositions and theories which were generally accepted as descriptions of some facts were expected to have the following characteristic features:

(i) To be communicable, i.e. to have a public meaning.

(ii) To be coherent with the previously existing body of knowledge in one of the two following senses: (a) to be either consistent with it, or, if inconsistent with some abstract, mainly non-referential expressions, to enable its referential expressions to be reorganised. This new organisation is preferable, either if it is of a simpler form than the previous one, or, what is much more important, if it gives a better explanation of past experiences, even accounting for those which were previously treated as exceptions. (b) To be either deduced from it, or to be used as a foundation for its definition.

(iii) To entail rules for predicting some new, previously unknown, future experiences. Practical actions undertaken in order to test the predictions should repeatedly and intersubjectively be successful.

If we now assume that the symbol x in our general scheme "x is true if and only if p" means the name of a proposition p and p satisfied these three conditions (which coincide with the previously mentioned criterion of objectivity):

(i) to be publicly meaningful,

(ii) to be coherent with previous knowledge (or to be theoretically supported),

(iii) to be verifiable; what we have got is the general scheme for the definition of "empirical" or "factual" truth.

Whenever we have a synthetic and true sentence as x, what stands as p in our scheme satisfies the criterion of objectivity, and vice versa, any p which satisfies this criterion implies x being a synthetic and true sentence.

Before we proceed by looking for a definition of logical (or semantic or analytic) truth, it is necessary to clear up the following difficulty. It may be argued that we have not escaped circularity: we want to define the concept of logic on the basis of concept of truth. In spite of that our criterion of objectivity presupposes logic. We cannot construct concepts and propositions without logical procedures, and making predictions and verifications cannot be done without both the logic of deduction and of induction.

The answer to this objection is that here we have logic at two different levels. In giving an informal definition of truth we assume the implicit logic of our ordinary discourse and scientific inquiries ("logica utens", as it was called by C.S. Peirce). By means of such a definition of truth we want to define explicit logic ("logica docens"), and to select among various possible formal calculi those which can be accepted as formalised logical theories and systems. To put it more clearly: if various formal calculi are considered as object languages (syntax) and their interpretations (semantics) are meta-languages, then a general theory of logic which defines terms like "logic", "truth", "meaning" etc., must be built in a meta-meta-language.

~~This procedure is not only methodologically perfectly legitimate, but also it is the only one possible for our purpose. If we assumed any particular formal logical systems we should eventually get the formally exact and precise definition of truth relative to that system. The price for the~~

Syntax is concerned with validity of formulae, semantics with the L-truth (of sentences expressed by these formulae) relative to a given logical system. Both presuppose the assumptions of the selected formal system. The general logical theory is concerned with truth in general (truth of various semantical theories of L-truth inclusive). There (fore) it cannot presuppose any particular formal system but only those terms which underlie all rational discourse (e.g. "symbol", "experience" etc) and which belong to what was called the informal, implicit logic.

This procedure is not only methodologically perfectly legitimate, but also it is the only possible for our purpose. If we assumed any particular formal logical systems we should eventually get the formally exact and precise definition of truth relative to that system. The price of the formal exactness

~~formal exactness~~ would be the narrowness of the definition. We could not use it outside the logical system for which it was created, consequently, we could not use it as a criterion for evaluating which systems of symbols belong to logic. On the other hand, the foundation of the concept of truth of the implicit informal logic of scientific inquiry entails some vagueness, but vagueness is the natural consequence of generality. What is lost because of the lack of precision should be gained by the largeness of the range of applicability. At any rate the building up of the general concept of truth has to pass through several stages. The initial one is the construction of the general definition of truth for an informal logic and this is what we are trying to do. And this corresponds to what is going on in all sciences. Before unformalised concepts for a certain field of inquiry are built, no successful formalisation is possible.

The fact is that serious logicians, whose work really makes a contribution to logic, do construct the rules of their systems having in view the possibility of deriving true sentences from true sentences. Consequently they have some conception of truth previous to the construction of their systems and this conception is an informal one.

When one wants to build up a definition of truth relative to an incompletely formalised language in which the implicit logic of scientific inquiry must be expressed, one must be careful to avoid the well-known semantic paradoxes ("Liar" etc). Tarski pointed out that no definition of truth in the ordinary language is possible which does not lead to these

paradoxes. What follows is that our meta-meta-language should not be the ordinary language, although on the other hand, it cannot be completely formalised, and must not presuppose any other symbolic forms besides those which are generally accepted in scientific methodology. Therefore it may be regarded as a "incompletely formalised" language. In order to avoid paradoxes we shall use a simple devise: we shall add to all terms of our meta-meta-language a prefix to distinguish them from the terms of lower-level languages which have the same verbal expression, but not the same meaning. We shall use the symbol "C" (objective) as the prefix of our general term "truth". The semantic term "truth" which belongs to the object-language in this case (meta-language of syntax on the other hand) will have as prefix the symbol "L" as usual. The term "truth" in an empirical meaning will be prefixed by "E". Then we shall postulate that no symbol of our language can be the name of a sentence which contains any O-term. Therefore, paraphrasing Tarski's example,[1] we can only get sentences like: "C is an O-true sentence if and only if C is not an L-true (or E-true) sentence" where C (is identical with the expression) "C is not an L-(or E-) true sentence" and this is not a contradiction, because here we have two levels of discourse and two different meanings of truth. C may not be empirically true; however, being logically true, it would also be O-true. And conversely C may not be logically true, however being empirically true it would also be O-true.

Now we come to discuss the definition of logical truth. Logical truth is usually understood as the truth of a sentence on the basis of the experienced logical rules solely, no matter what facts are. In other words, a proposition is logically true (analytic) when its truth depends solely on the definitions of the symbols it contains (on the meanings of its constituents.)

[1] See: Tarski, op. cit. S. 271.

This definition is too vague and weak and allows such applications as lead to really paradoxical results. So e.g. from a conventionalist point of view an arbitrary logical system can be constructed containing rules incompatible with the rules of inference used in ordinary discourse and in science. One of these queer rules might be represented for example by the following scheme:

$$\frac{\vdash (\exists x) fx \supset (x) fx. \quad \vdash (\exists x) fx.}{\vdash (x) fx}$$

If the symbol "$(\exists x)$" was given the ordinary meaning of existential quantifier and "(x)" the ordinary meaning of universal quantifier, we should be in a position to accept as "logically true" all those propositions (mostly false) obtained by generalisation from particular to universal without any supporting evidence.

Here we have one more argument that the choice of rules and definitions in a system claiming to be considered as logic cannot be a matter of arbitrary convention and subjective disposition. Or vice versa, if that was the case, the obtained system would be perfectly uninteresting for logic and any science, except may be for psychology, which might study the motives that led someone to construct such and such a system.

However, much work has been done recently to find a more precise definition of logical truth such as to prevent all possible conventionalist extravagances and to bridge the gap between factual and logical truth.

Carnap has shown recently that a number of equivalent and more specific definitions of logical truth (L-truth) of a sentence (Si) of a certain language L can be constructed.

(i) The open logical formula corresponding to Si (e.g. '$f_x \vee \sim f_x$' is universally valid (i.e. satisfied by all values of the free variables). (Here it is presupposed that t contains corresponding variables for all descriptive constants.)

(ii) The universal logical statements corresponding to Si (e.g. '$(f)(x)(f_x \vee \sim f_x)$') is true. (Here it is presupposed that L has variables with quantifiers corresponding to all descriptive constants.)

(iii) Si is satisfied by all values of the descriptive constants occurring.

(iv) Si holds in all state-descriptions. (A state-description is a conjunction containing for every atomic statement either it or its negation but not both, and no other statements. Here it is presupposed that L contains constants for all values of its variables and, in particular, individual constants for all individuals of the universe of discourse.)

"Each of these formulations presupposes, of course, that rules for the system L are given which determine the concepts involved, e.g. rules of formation (determining the forms of open formulas and statements i.e. closed formulas), rules for the range of values of all variables and

for (iii) also analogous rules for the range of values for all descriptive constants and for (iv) rules determining those state-descriptions in which any given statement holds. Form (iv) is quite convenient if L has the required form. Form (iii) imposes the least restrictions on L." [1]

In these four definitions a connection between logical and descriptive expressions is established, (descriptive symbols being understood as the symbols conveying some factual conformation or describing some empirical content). The idea is put forward that the descriptive expressions constitute the values of the variables of logical formulae and that such a logical formula is considered as logically true if it is satisfied by all the values of its variables. Or, on other words, for a sentence to be logically true means to be a true statement in all cases when descriptive expressions are substituted for variables (ii), provided that each particular decision((a) whether an expression is a sentence, (b) what are the ranges of variables, (c) what are the ranges of descriptive constants) is regulated by explicit rules.

If this is what we mean by the logical truth of a sentence, then obviously it is not independent of the experience embodied in the meaning of descriptive constants in the sense that it has nothing to do with them and that it might be still be logically true even if all the facts were different. The expression "independent" is very ambiguous here. For, in order to see what is meant by speaking about the independence of logical truth of empirical knowledge, we shall analyse the following

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1. Rudolf Carnap, "Meaning Postulates", Philosophical Studies, vol.III, No.5 Oct.1952, p.67.

example:

Suppose we have the formula " $f(x) \supset g(x)$ " and the following semantical rules are given:

1. " $\supset$ " means "implication" i.e. connective between two expressions such that either the antecedent is false or the consequence is true.
2. "f" means "planet".
2. "g" means "rotating round the sun".
4. range of (values of) x contains: Mercury, Venus, Earth, Mars, Saturn, Jupiter, Uranus, Neptune and Pluto.

On the basis of these rules the (formula) " $f(x) \supset g(x)$ " is logically true. „Being a planet implies" „rotating around the sun" in accordance with the very meaning of the words used. Now, the truth of this sentance is independent of our empirical knowledge about the planets, not in the sense that the sentence would still be true even if Mercury, Venus, etc stopped rotating around the sun, but in the sense that it is true for all of them, both that they are planets and rotate round the sun, so that it is irrelevant which one of them we shall pick up as the value of the variable x. Now if we discover that Pluto in fact does not rotate around the sun, we can either drop the formula "f(x) $\supset$ g(x)" altogether or change the semantical rules. In the first case we do so because, with the previously given meaning of x, it is no longer true. In the second case we still hold the formula to be logically true, but, with the changed meaning of x.

it now expresses a new proposition. In both cases it is evident that the logical truth of the proposition expressed by our formula has been affected indirectly ④ by the discovery of a new fact.

In the same way the formula " $\sim (Fx.\sim Fx)$ ", ~~presumably~~ where x is supposed to have an unlimited range of values, is logically true independently of empirical knowledge, not in the sense that it would still be true even if we found some objects having and at the same time not having certain properties. It is logically true only in so far as we agree that there are no such objects and postulate by a semantic rule that x has an unlimited number of values. Then the formula is independent of our empirical knowledge only in so far as it is irrelevant which one of the infinite range of objects we take as the value of the variable x.

One might object to this argument that " $\sim (Fx . \sim Fx)$ " is simply our linguistic device and that, as the meanings of all its constituent symbols are simply a matter of our arbitary choice, " $\sim (Fx . \sim Fx)$" will remain true for ever. And the argument may go on: Even if we suppose that our scientific knowledge about the world can change so profoundly that we shall come to the conclusion one day that everything develops so quickly that each object has and at the same time has not all its properties, this cannot affect the truth of our formula. This formula is the result of our creation, part of our language, and it keeps its truth-value unless we decide otherwise.

And to this it should be answered: under the given assumption of a profound change in our picture of the world either (a) we shall abandon our

④ New empirical facts are indirectly relevant for an analytic proposition in so far as they directly only induce us to change the meanings of our words. However with new meanings of their terms the statement previously believed to be analytical are no longer true. E.g. the statement "Atoms are indivisible parts of matter" was an analytic statement so far as the term "atom" was at first taken to mean "indivisible particle". The discovery that the referents of this term are in fact divisible directly caused our change of the meaning of the word "atom" and indirectly made us to reappraise the truth-value of reinterpreted sentence "Atoms are indivisible parts of matter".

formula, or (b) we shall change the meanings of our symbols, still keeping the linguistic expression, but changing the proposition which it asserts, or (c) we shall change nothing in our syntax or our semantics, and we shall be faithful to the linguistic device we have created, keeping our formula with the same constituents, same structure and same meaning. But it would be a Quixotical attitude: our formula would no longer be philosophically and logically interesting, because it would constantly mislead people. Whenever they substituted any descriptive (referential) expressions for x they would get a factually false statement. They would be completely confused by the strange fact that logical truth was apparently a powerful instrument for discovering factual falsity and vice versa. And of course this is the very opposite of what we expect to get from logic.

Here obviously the attitude (b) would be preferred by the defenders of any formula previously considered as logically true and later found incompatible with some empirical facts (in so far as the old interpretation was kept).

So the critics of the "principle of contradiction" may enumerate some types of fact where the principle apparently does not hold, e.g. (a) cases of quick transition from one state to another, or quick qualitative or quantitative changes, like in the sentence "This chemical element is and it is not actinium A. (the half-value period being only 0,002 sec.); (b) cases of having one property in one relation or respect and not having it in another, as in the sentence "Mr X is brave (meaning but not

saying: in the war, in fighting) and Mr X is not brave (meaning but not explicitly stating: when facing his wife or when sick, etc); ∫(c) borderline cases when it is not clear whether an object is a member of a class, or is not, as in the sentence: "Mr X (born of an English father and an American mother, having dual citizenship and living in both countries) is an American and is not an American." ∫(d) in the case when very general and abstract terms are used without the qualifications necessary to make the meaning of the sentence clear, e.g. "A moving body is and is not at a certain place at a certain time."

The defenders of the principle of contradiction try to meet these criticisms by adding certain qualifications to the principle, e.g. temporal and relational qualifications, and by putting such demands as that the range of application of the terms used must be fixed, that the principle can not be applied to vague terms, etc. The effect of these qualifications is the change of the range of the variable x in the formula " $\sim (F_x \cdot \sim F_x)$ " or " $\sim (p \cdot \sim p)$ ". Instead of allowing an unlimited range, now we have excluded from the range of x all those terms whose temporal and relational perimeters are not given, whose meaning is vague and whose extension is not fixed. Now we get no contradictions because all the sentences used as examples against the principle of contradiction can be reformulated as follows:

(a) This chemical element is actinium A in t_1 but is not actinium A in t_2 (say 0,01 sec. later).

(b) Mr X is brave in war, but not brave when sick.

(c) Mr X is an American in so far as his mother was American, and he is an American citizen, but he is not an American in so far as his father was English and he is a British citizen.

(d) A moving body is in an infinitely small interval of time t_1 at the point p_1 but in the next infinitely small interval of time t_2 it is not at p_1.

However, the important point is that the formula " $\sim(Fx \cdot \sim Fx)$ has not the same meaning i.e. does not express the same propositional function with the qualifications and without them. We have changed the relevant semantical rules (in particular the rules determining the range of the variable x) therefore the formula " $\sim(Fx \cdot \sim Fx)$ " is not logically true in the same sense. Without the restrictions in the range of variable x the formula is no longer logically true. With these restrictions or - if we avoid ordinary language as the object-language - in a suitable object-language with suitable semantic rules, the formula is logically true.

Now we come to the problem of the relation between the two aspects of truth

What follows from the previous discussion ~~about~~ empirical and logical truth is that although they should be clearly distinguished as concepts, there is no cleavage between them, and the same propositions can be both empirically and logically true in different contexts. The relation between these two is best represented by the logical connective "disjunction" in its ordinary meaning.(1) ? ?

But you yourself have just said that within one theory [illegible] about the same propositions

There are the following three possibilities for a sentence x to be

(1) p is objectively true $=_{Df}$ p is empirically true or p is logically true $=_{Df}$ p is either empirically true, or logically true, or both empirically and logically true.

regarded as objectively true:

(i) x is only empirically (E-) true but not logically (L-) true, which implies, in distinction to L-truth, that (a) it is supported by some theory but not necessarily proved within some formal deductive system; (b) it is verified in a practical way.

(ii) x is only L-true but not E-true, which implies either that (a) x expresses a rule such that it can not itself be transformed into an empirical statement, but it can be used for getting empirical statements, and the problem of truth arises only in so far as whether x really expresses a rule by means of which we obtain empirically true statements. Or (b) although applicable, x has not been actually applied, therefore, it cannot be asserted as a universally true statement (true for all values of its variables).

(iii) x is both E-true and L-true. That implies one of the following two: (a) x is firstly proved within some unformalised theory and verified as an E-true statement. Then it is either taken as an axiom of a formal deductive system, or it is proved in it. This happens whenever we formalise a theory which has been already established as empirically true. In relation to each proposition of the theory, we make here the step from the E-truth of "(x)(...x...)" to the L-truth of "(...x...)".

(b) x is firstly constructed as an L-true formula, then it is found E-true for all values of its variables and expressed as a universal empirically true proposition.

Here we make the step from the L-truth of "(...x...)" to the E-truth of "(x)(...x...)".

The third possibility shows that there is no gap between logical and empirical truth. In fact (iii) is the ideal case for all formal systems.

Formal systems which could neither be empirically applied nor are applicable should be considered only as preliminary work in the best case. There is an analogy between such sterile formal calculi and sterile experiments. Neither of them is necessarily uninteresting because sometimes even walking along blind alleys can be of tremendous importance in science. However, sterile calculi in the same way as sterile experiments do not remain as results, as achievements, but as indications of a negative experience.

Finally, something remains to be said in connection with the old problem: How to formulate ~~definitions of truth and conditions of~~ propositions about truth which should be true themselves. ~~Many philosophers thought that~~ It looks as if either there must be some circularity in any attempt to solve the problem or the problem must be dismissed as insoluble (or meaningless). The truth of what we say about the truth must be postulated.

However, since the discovery of the fact that in order to avoid semantic paradoxes we must build up a hierarchy of languages the problem has become solvable. All propositions in which the predicate "true" appears are relative to a certain language L, and can be formulated

only in a higer-level language. If we take a formal calculus C as an object-language, none of its formulae can contain the expression "logically true". The semantics of C provide a meta-language in which "L-true" can be used as the predicate of formulae of C. Now in order to discuss various semantical conceptions of truth we need a general logical theory, and we must construct a meta-meta-language for it. Within the framework of this language we can attribute the predicate O-true (objectively true) to the various semantic expressions. When now the question arises of the truth of the propositions of a general logical theory, it can be settled only be constructing a language of the next higher level.

This process is not circular, neither is it an infinite regress, for instead of the constant recurring of one and the same concept of truth, concepts of various meaning and generality are being built up.

Chapter IV

1. Definition of logic. Science, logic, Meta-logic and Epistemology.

The purpose of the discussion in the previous chapters was to explain the most fundamental concepts which are necessary for the clarification of the concept of logic. Now that a certain attitude has been taken in relation to the problems of "meaning" and "truth", the construction of the definition of logic and establishment of the relations between science, logic, meta-logic and epistemology is to a large extent a matter of analysis. However, in order to get a simple and formally correct definition, it is necessary to construct a number of auxiliary definitions, and they will be interwoven into a sequence of definitions ~~which have already been ever used in a more or less informal way. In this way we shall get the following sequence,~~ where a certain number of fundamental epistemological terms plays the role of the undefined terms of the language.

D$_1$. Objective Experience = Df. the constant features of experience of a group of people under some given conditions C in a certain interval of time T.

D 1-2 Objective = Df. relating to objective experience.

D 1-3. To correlate = Df. to bring into a certain relation each member of one class with a certain member of an other class.

D 1-4. x expresses at least indirectly O-experience = Df. x is a linguistic[1]. symbol which correlates constant features of experience of a certain group of people either directly or through some other symbol to which it stands in some specifiable relation.

D 1-5. To communicate = Df. to use a symbol which can express O-experience of all people of a certain group.

D 1-6. x is O-communicable = Df. x is a symbol which expresses at least indirectly O-experience of a certain group[2]. of people. (Or x is a symbol which enables all people of a certain group to communicate.)

D 1-7. x is O-descriptive = Df. x is an O-communicable symbol which expresses O-experience of a certain group of people in a direct way. (Being O-descriptive entails being O-communicable; or, O-descriptive symbols are a sub-class of O-communicable symbols.)

D 2-1. O-rule = Df. a disposition, common for a certain group of people, to use a certain symbol in a certain way.

D 2-2. x is a concept[3]. = Df. x is a set of O-rules for the use of an O-communicable term.

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1. It is necessary to specify that we are dealing here only with symbols used in natural and artificial languages. Otherwise O-experience can also be expressed by pictures, music, etc.
2. The group in question might be specified by an index of the prefix O. e.g. "O_g" would specify a symbol as communicable for the group of all logicians.
3. The concept here defined is an O-concept as is evident from the definition. However, for the sake of the simplicity of language we omit the prefix O here and in the following definitions. It should be borne in mind that all terms used in this language are O-terms.

D 2-21. x is a concrete concept = Df. x is a set of O-rules for the use of an O-descriptive term and for the selection and classification of practically relevant experiences.

D 2-22. x is an abstract concept = Df. x is a non-concrete concept.

D 2-3. x is a proposition = Df. x is a meaningful and assertible connection[1] of concepts.

D 2-4. x is an inference = Df. x is a sequence of propositions such that from one or more propositions (premises) a new proposition (conclusion) follows.

D 2-5. x is ordered = Df. x is a set of elements such that all elements of the set stand in a certain relation to each other.

D 2-6. A theory = Df. an ordered set of inferences.

D 2-7. An axiom = Df. a proposition which is not inferred from the other propositions but which can be used as a premise for inferring all the other propositions of the given theory except the other axioms.

D 2-8. A consequence = Df. a proposition which is inferred from a certain set of axioms.

D2-9. x is provable = Df. x is either an axiom or a consequence of a certain theory.

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1. The usual connectives - negation, implication etc. and universal and existential operator are concepts as well. Therefore the expression "connection of concepts" gives account of what any sentence expresses.

D 2-10. x is a deductive system = Df. (1) x is a sub-class of the class of all propositions; (ii) x is the conjunction of the class of axioms A and the class of consequences which can be inferred from A.

D 2-11. x is theoretically supported = Df. x follows at least indirectly from some theory or (in a special case) x is a provable proposition in a certain deductive system.

D 3-1. x is directly verified = Df. x satisfies the following conditions: (i) x is a proposition expressed in descriptive symbols; (ii) to x there corresponds a rule for the prediction of some particular O-experience Ex under conditions C at the time-interval T; (iii) when we create condition C at the time T we experience Ex.

D 3-2. x is indirectly verified = Df. x satisfies the following conditions: (i) x is an O-proposition (ii) in conjunction with certain propositions x entails a set of directly verified consequences which are not deducible from the other premises alone, (iii) these other premises are theoretically supported or directly verified.

D 3-3. x is verified = Df. x is at least indirectly verified.

D 3-4. x is verifiable = Df. if certain conditions C at the time-interval T could be created x would be verified.[1]

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1. The term "verifiable" is taken sometimes in a wider sense including both possible verification and falsification. As it should be used here for the definition of truth it is taken to mean only positive possible verification.

D 4-1. x is an objective state of affairs[1] = Df. there is a concept x_c which is the constituent of the class K of theoretically supported and verified propositions and the following implication is postulated: if K is a class of theoretically supported and verified consequences, then there is a certain x which exists independently of anyone's experience and whose characteristics are approximately described by x_c.

D 4-2. y denotes x = Df. y is a class of communicable symbols which describes approximately a class of objective state of affairs x.

D 4-3. x is a particular object = Df. there is a communicable symbol y and y is a proper name and y denotes x.

D4-4. x is a class of objects = Df. there is a communicable symbol y and y is a class-symbol and y denotes x.

D 4-5. x is a property = Df. there is a communicable symbol y and y is an one-termed predicate and y denotes x.

D 4-6. x is a relation = Df. there is a communicable symbol y and y is a many-termed relation and y denotes x.

D 4-7. x is a fact = Df. there is a communicable symbol y and y is a sentence and y denotes x.

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1. The term "objective state of affairs" is taken here in a narrow sense of objects, facts, properties etc. to which our concepts and propositions adequatly refer, i.e. in the sense of objects etc. of knowledge. In a wider sense an objective state of affairs is anything that exists independently of human mind, no matter whether it has ever been discovered and described.

D 5-1 x is synonymous with y = Df. x and y perform the same function in communication i.e. x and y correlate the same O-experience of a certain group of people.

D 5-2 To define x = Df. to construct a set of sentences which consists of two expressions connected by the sign of identity such that the expression on the left is x and the expression on the right is synonymous with x.

D 5-3 Intension of x = Df. concept (or a proposition) which is expressed by the symbol x.

D5-41 Extension of a descriptive symbol x = Df. the class of objective states of affairs which are denoted by x.

D 5-42 Extension of x (extension in general) = Df. the class of cases (the range) determined by the intension of the symbol x (to which x can be applied).

D 6-1. x is an explicit sense of y = Df. x is a definition of a symbol y in terms of a set of O-communicable symbols.

D 6-11. x is an implicit sense of y = Df. (i) there is an O-concept (proposition) expressed by the symbol y (the intensional sense of x); (ii) there is a class of cases to which the symbol y can be applied (the extentional sense of x).

D 6-12 x is the sense of y = Df. x is at least an implicit sense of the symbol y.

D 6-2 x is an explicit reference[1] of y = Df. x is a definition of

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1. The term "explicit reference" may sound odd, however it is clear from the definition that it means "explication of reference" i.e. a linguistic description of the objective state of affairs which y refers to.

the symbol *y* in terms of O-descriptive symbols.

D 6-21. *x is an implicit reference of y* = Df. *y* is a class of descriptive symbols which denotes a class of objective states of affairs *x*, and the following specification holds:

(i) if *y* is a proper name, *x* is a particular object

(ii) if *y* is a class symbol, *x* is a class of objects.

(iii) if *y* is a one-termed predicate, *x* is a property

(iv) if *y* is a many-termed predicate, *x* is a relation.

(v) if *y* is a sentence, *x* is a fact.

D 6-3. *x has a meaning (x is meaningful)* = Df. *x* has a sense or has both a sense and a reference.[1]

D 6-4. *To designate* = Df. the relation in which a symbol stands to its sense.

D 6-5. *To denote* = Df. the relation in which a symbol stands to its reference.

D 6-6. *To mean* = Df. to designate or both to designate and to denote.

D 7-1. *x is a semantic rule for y* = Df *x* is a rule which states explicitly at least one part of the meaning of the symbol *y*.

D 7-2. *x is a complete set of semantic rules for y* = Df. *x* is a set of rules which states explicitly the full O-meaning of the symbol *y*.

D 7-3. *y is interpreted by x* = Df. *x* is a complete set of semantic rules for the symbol *y*.

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1. As all the terms previously defined, including "sense" and "reference" are O-terms (related to "objective experience") the definition of meaning is concerned with the O-meaning (public meaning).

D 8. x is applicable to y = Df. y is a class of descriptive symbols and x is a set of rules which either (i) explain the meaning; or (ii) furnish a scheme of inference; or (iii) lay down the conditions of verification for all the members of y.

D 9. x is empirically (E-) true = Df. x is a communicable symbol which expresses a proposition p and p satisfies the following conditions: (i) p is meaningful; (ii) p is theoretically supported; (iii) p is verifiable.

D 9-2. x is logically (L)true = Df. x is a set of communicable symbols which satisfies the following conditions: (i) x is interpreted; (ii) x is provable (i.e. either an axiom or a consequence in a deductive system; (iii) x expresses a rule which is applicable (to a class of descriptive symbols).

D 9-3. x is objectively (O)true = Df. x is E-true or L-true.

D 10-1. x is an O-truth condition for T = Df. x is a set of rules which all propositions in the theory T must satisfy in order to be appraised as O-true.

D 10-2. Logic = Df. the class of all theories which lay down the O-truth conditions for any O-theory.

Or, in other words:

D 10-3. Logic = Df. the class of theories expressed by symbols which satisfy the following conditions: (i) they are O-interpreted; (ii) they are O-provable; (iii) they are O-applicable (i.e. they express a set of rules which either explain the meaning or

provide a scheme of inference or lay down conditions of verification for all the numbers of a certain class of descriptive symbols).

2. Logic and Special Science - General and Special logic.

In view of the given meanings of the terms used, it is obvious that this definition of logic rules out both extremes: the unlimited freedom of constructing numberless systems of completely arbitrary "private" logic, and also the dogmatism entailed by the thesis of one and unique absolute logic which leads us to the discovery of absolute truth.

However, it might appear to be too wide and to fail to distinguish between logic and some class of empirical theories belonging to a special science and expressed in the form of an axiomatic system. The argument might go like this:

If we take, for example, Newton's formula of the law of gravitation, it is obviously a propositional function proved in Newton's Mechanics and it is interpreted and applicable. When we substitute some actual parameters for variables m_1, m_2 and r in the formula $F = g.m_1.m_2/r^2$, we shall get an O-true statement. Then it follows that Newton's formula should be conceived as a logical rule and even as a law of logic, because a logical rule is only an operational expression of a logical law. This sounds odd and seems to lead us into a confusion of

logic and empirical science.

However, no confusion should arise if we bear in mind what O-applicability means (for all descriptive constants from the field referred to by a variable of the given formula, when we insert any of them into the formula as the value of the corresponding variable, an F-true sentence must be obtained). Now, the vast majority of empirical laws are not true for all values of their variables; they allow various exceptions and hold only under certain conditions which are not specified in the formula expressing the law. But if they are O-true for all values of their variables, they should be considered as logical principles (laws, rules) in the sense of being constituents of the special logic of the given empirical sciences, no matter whether this logic is already constructed as a whole system.

And there is nothing odd in this talking of logical principles in this special sense, nor/in talking of the special logic of some empirical science or of some particular field of philosophy.

The word "logic" is very often used in this and many other special senses in ordinary discourse, in science, and even in philosophy.

In high-brow newspapers like the Observer we may very often find sentences like this: "What both sides must do is to learn to compete politically and economically, while avoiding

military policies that are either futile or contain the risk of accidental suicide. That is the logic of the world situation."[1.]

Some special logical systems in science have already been constructed since the beginning of this century. Mathematical logic is in fact a special logic, although the most general of all possible special logics in so far as mathematics can be used as an instrument in many other special sciences. To say that it is only a special logic amounts to saying that it is not universally applicable, or in other words, that there is a limited range of terms which can be used as the values of their variables in order to get O-true sentences. E.g. if we take as values of the variables the concepts with blurred boundaries the open formulae of any system of mathematical logic might turn out to be false sentences. However, most concepts used in empirical sciences, particularly in the immature ones, are rather vague. At least such is the state of affairs at the present time. The tendency of the development of empricial sciences indicates that a time might come when the language of all sciences would become as precise and exact as the mathematical one and when Leibniz's dream of a "Mathematica/ universalis"would be realised. However, such a state of science really cannot be anything more than a limit-notion.

In the last two decades there have been attempts to construct special logical systems corresponding to some particular

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1. "Comment", The Observer, 22 iv. 1956, p.8.

branch of a given empirical science such as "logic of quantum mechanics" by Birkhoff and Neumann[1], Louis Destouches and Paulette Fevrier[2], "logic of the cell theory" by Woodger[3], etc.

Philosophers also use the term "logic" in various special senses. I shall give only a few illustrations, chosen at random. Wisdom for example is very fond of talking about the logic of other minds.[4] Ryle in his Dilemmas makes a comparison between the inquiries which belong to formal logic and those which belong to philosophy. He comes to the conclusion that whereas "the formal logician really is working out the logic of and, not, all, some, etc. the philosopher really is exploring the logic of the concepts of pleasure, seeing, chance, etc."[5] Price in Thinking and Experience devotes a whole chapter to "the logic of sign-cognition"[6].

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1. T. Neumann and G. Birkhoff, The Logic of Quantum Mechanics, Annals of Mathematics, vol.37, p.823.

2. Paulette Destouches-Fevrier, "Logique et Theories physiques" XV Congres international de philosophie des sciences, Paris 1943, II logique, p.45 Paris 1951.

3. Woodger, The Axiomatic Method in Biology Cambridge 1937
The Technique of Theory Construction, Encyclopedia of Unified Science, vol. II, no.5. Chicago 1939

4. John Wisdom, Other Minds, Oxford, 1952.

5. Gilbert Ryle, Dilemmas, Cambridge, 1954, p.119.

6. H.H.Price, Thinking and Experience, London 1953, pp.125-144.

This way of talking is not peculiar to British philosophers. From the volume of "The Library of Living Philosophers" devoted to Dewey we learn that Dewey used to give courses in the logic of ethics[1] and that his work Experience and Nature is "the logic of common experience considered in terms of widest and most inclusive generality."[2]. Of course one would expect all sorts of logic from such an extremely original logician as Peirce. Thus he had logics of evolution,[3]. of history,[4]. of continuity,[5]. of quantity,[6]. of events,[7]. of mental operations,[8]. etc.

The conclusion should be drawn therefore, that the idea of a social logic is in accordance with the widely extended habit of making a distinction between a very general meaning of the word "logic" referring to the whole of human experience, science and philosophy, and a special one referring to one particular field or theory or one single concept.

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1. The Philosophy of John Dewey, ed. Schilpp, vol I.p.32.
2. Ibid, p.72
3. C.S. Peirce, Collected Papers, Cambridge 1935, vol.VI.para.218.
4. Ibid, vol. VI, para 512
5. Ibid, vol.III, para.617
6. Ibid, vol, III, para.526.
7. Ibid, vol,VI, para, 218
8. Ibid, vol.IV, para. 539

The main difference between general and special logic is in the range of applicability.

In general logic the range of any variable of a certain type consists of all the descriptive constants of the same category (individual, class, ~~property~~ predicate, dyadic-, triadic-, etc relations, etc.) no matter to which field of knowledge the constant refers.

On the other hand, the characteristic of a special logic is: besides these variables with a universal range of application, there are also variables whose range consists of all the descriptive constants of the same category which belong to that limited field of knowledge to which the special logic refers.

The important point is that this distinction between general and special logic does not blur the distinction between logic as a whole and the special sciences.

The relation between a certain empirical science and the special logic of that science is always - linguistically expressed - the relation between an object-language and the corresponding meta-language. That means that the task of a logical theory, no matter how special it is, is always to lay down the truth-conditions for the science to which it refers. And this laying down the truth-conditions ~~comprises~~ comprises: (1) the clarification of the concepts of the science in question

(the establishment of the "rules of designation" or "meaning-postulates" in Carnap's formulation); (ii) the explanation of its basic propositions and rules of inference; (iii) the explanation of the rules of verification.

Now we can briefly explain the difference between general and special logic by saying that the object-language of the former is the whole of science,[1.] whereas the object-language of the latter is the language of a particular science, or a branch of science, or even only a particular scientific theory.

The expressions of any of these object-languages are "empirically-true" or "empirically-false", but the language in which they can be appraised as such, i.e. the language which contains the predicates "E-true" and "E-false", cannot be the language of the empirical science itself, but the language of logic. An empirical truth by definition entails the fulfilment of the three main conditions: O-communicability, O-provability[2.] and O-verification. It is obvious that the formulae of the logical theory of any empirical science must provide the corresponding three kinds of criteria, and make possible our decisions in three respects:

(i) which terms of the theory have an objective meaning and can be assessed as O-communicable.

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1. If this "whole of science" were expressed in a most general way, the obtained theory would be a scientific ontology.
2. In order to simplify our terminology "provability" is here taken in a weak sense of "being supported by a certain theory".

(ii) Which propositions of the theory are accepted on the basis of some theoretical considerations (in the special case, which propositions are provable in a deductive way).

(iii) Which propositions can be considered as verified.

In order to establish these criteria, any logic referring to an empirical science must contain three parts: (i) a theory of meaning (semiotics, or semantics of the given special science); (ii) theory of proof, whose important part is constituted by deductive logic, and (iii) theory of verification which is usually dealt with in the text-books of logic under the heading of "Methodology of Science".

For obvious reasons the logic of mathematics need not contain the theory of verification, but it is not so for general logic.

It is not difficult to see the similarities and differences between the sentences of the logic of (analytic sentences) and sentences of an empirical science (synthetic sentences). Both must be O-communicable and O-proved[1], but whereas the former must be O-applicable, the latter must be O-verifiable. In other words, a sentence of logic is only a scheme, a general structure for an empirically true sentence, such that by appropriate substitution of descriptive constants we may actually get E-true sentences.

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1. Forms of proof may be very different in these two cases.

Empirical (synthetic) sentences, on the other hand, must be already verified in order to be assessed as true, or verifiable in order to be considered as hypotheses, or finally, falsified, in which case they would be dropped from the theory. Therefore they are not only rules i.e. structures and schemes for statements, but statements themselves.

This difference is described by saying that the former are logically true, whereas the latter are empirically true. We have seen that this difference allows some overlapping, because universally true empirical statements can be transformed into open logical formulae and treated as logical rules of the particular science in question, and vice versa open logical formulae may be actually verified, found to be universally E-true and expressed as universal empirical statements.

3. Logic, meta-logic and general theory of logic.

Now, when logic is so conceived as a meta-theory of science, meta-logic becomes the meta-meta-theory of science. Its task is to lay down the conditions of the L-truth of logical expressions. As logical truth, by definition, entails the fulfilment of the three main conditions (i) O-interpretability, (ii) O-provability and (iii) O-applicability, the expressions of meta-logic must obviously provide the three corresponding criteria and enable us to make decisions in the three following respects:

(1) Which expressions of the logical theory in question are objectively meaningful.

(2) Which formulae are provable; and

(3) Which formulae are applicable in so far as the logical theory as a whole is applicable.

In order to establish these criteria any metalogical theory referring to a particular logical theory must contain three parts; (i) Semantics of L (theory of meaning); (ii) Metalogical theory of proof for L; and (iii) Pragmatics of L (theory of application).

Most of the work on ~~meta~~ meta-logic has been done in the field of the theory of proof (the axiomatisation of logic, decision problem, proof of the consistency of logic, etc.). Semantics has been developed only in the last ~~two~~ three decades, mainly by Tarski and Carnap. Semantics, as it has been formulated so far, does not contain the whole of metalogic. Besides providing definitions of meaning for the logical terms, it is also assumed that its task is to lay down the truth conditions of ~~a~~ a given logical theory. What kind of truth is here in question? It is not logical truth, in the sense in which we have defined it. If laying down the truth-conditions is a matter of convention, which is the impression one gets from studying Carnap's work, then we might get countless semantic theories with countless sets of "truth-conditions" and then we need some stronger criteria in order to decide which of these really are "the conditions of logical-truth". Such a criterion seems to be available by a suitable theory of the

applicability of logical calculuses. However, this is the part of metalogic which is badly neglected and where theoretical considerations are almost completely replaced by intuition and trial-and-error methods.

A general theory in which the problem "What is logic (and metalogic)?" is discussed and the definitions of meaning, objective truth and logic are given is a higher-level theory in relation to metalogic. Such a general theory of logic should provide criteria as to which formal calculuses and their interpretations belong to logic. This theory would be a part of the general theory of knowledge, whose task would be to investigate the process of knowledge in its entirety - discursive rational thought being only one part of it. A special language should be constructed for the theory of knowledge, in order to avoid confusing the levels of discourse. The sentences of the theory of knowledge would have to be true in an even more general sense than O-truth.

~~Chapter V~~

~~Form and Content, Abstract and Concrete Logic~~

~~1. The controversy between formal and non-formal logicians~~

4. Formal and non formal logic

Once it is accepted that logic stands in a particular functional relationship to science and that its expressions must be applicable to a certain theory T in the sense of providing truth conditions for it, it follows that it cannot be purely formal in the sense of being absolutely independent of the context of T.

If for an alleged logic to be formal means to "hold" or to be "true", regardless of any theoretical content to which at least in principle it can be applied, then either this allegation is not true because the given logic is not formal in that sense (it may be in some other), or such a logic is philosophically insignificant and one may wonder if it should be called logic at all. Therefore no logic is formal in that sense.

On the other hand, if the term "formal" refers to the constant and structural features of a certain theoretical context, then no logic is possible which is not formal in that sense. It always lays down the truth conditions for a context C as a whole

and must abstract from the particular sub-class context within C to which the truth conditions should be applied.

Now it is always possible to construct a more concrete, or more special logic applicable to some of these sub-class contexts. It may be obtained from the more abstract and general system by adding some further truth-conditions specifically for the more narrow context of application. However, by this procedure we have only changed the system of reference. Our "concrete" logic will again be constructed only in view of the constant features of the new system of reference. In other words, it will again be formal relatively to another, more narrow, system of reference.

Such a solution would settle the old dispute between formal and non-formal logicians.

The adherents of formal logic have rightly noticed the invariant character of logical rules. These rules express some relationships which do not change from case to case and for which the circumstances of the particular case of application must be irrelevant. However, they have been inclined to overlook the fact that the nature of logical rules depends on the constant features of the theory, or science as a whole, for which they are constructed. Thus they have absolutised logical forms and created the illusion that they hold a priori.

On the other hand, various critics of formal logic from Hegel onwards have rightly noticed the connection between logical forms and theoretical content. Failure to take into account specifities of the given context of application can really result in grave mistakes. In many cases mere observance of few general logical rules does not guarantee that we shall derive objectively true propositions from objectively true propositions (e.g. if the latter are O-true under some conditions which have been neglected in our particular inference).

However, they have been inclined to overlook the fact that if logic is to be constructed as a science, it cannot help taking into consideration for the formulation of its rules only constant features of a context to which it is correlated. Many specific features of context must be dropped out, and to demand otherwise would be a perfectionist point of view. Therefore there are three alternatives:

A concrete logic is either (i) a limit-concept or (if it should be expressed as a meta-theory and embrace the whole context of a theory) it is (ii) a special formal logic (specially constructed for a given context of application) or finally (iii) it is not a theory at all, but a set of intuitively chosen procedures.

I think that the second alternative should be accepted.

However, in that case it appears that the difference between formal and non-formal logic is relative to the system of reference, and consequently to the degree of generality and abstraction. A logical theory is more or less formal, abstract and general in relation to some other logical theory if it is applicable to a more or less extensive system of reference.

Assuming the existence of a hierarchy of logical theories of different levels, the same logic would be abstract and general in relation to some more special theory, and concrete in relation to some more general one.

In order to clarify this relation between formal (abstract) and concrete logic and to show that the difference between them is not absolute, we shall proceed by studying the concept of "form" and the relation between "form" and "content" in more detail.

5. <u>The meaning of "form"</u>.

The word "form" has an enormous variety of overlapping meanings . We apply it firstly to material objects when we want to say that some of them have the same shape. Then it occurs very often in art and literary criticism, when someone wants to say that a poem or a picture or a symphony has a particular pattern. Sociologists give it a rather similar meaning when they speak about the structures of social organisations. Psychologists refer sometimes to perceptions, thoughts, feelings,

etc. as forms of consciousness. Finally there is the particular meaning of the word "form" we are interested in, when it is applied to the conceptual and linguistic structures of science and logic.

In all these cases the commonsense meaning of form is something stable, constant, something that remains invariant through the flux of phenomena, and at the same time something which underlies various similar phenomena representing their general features. Thus we can discover the form of any particular class of things, or class of the successive states of the same thing, by neglecting all that is variable and changing from case to case and paying attention to those properties and relations which remain constant in the given context.

(a) Form of propositions.

By such a procedure, starting from the commonsense level without any previous knowledge, Greek logicians and particularly Aristotle succeeded in discovering a number of logical forms. By observing actual human speech they noticed that our most important statements, e.g. when we try to define something, consist of a subject i.e. expression which refers to what we speak about, the connective ("copula") "is" or "are" and the predicate i.e. the expression which refers to what we say about the subject.

The important thing to notice is that they had not only to find out which terms remain constant in all formulations of judgments, but also to establish one-to-one correspondence between the remaining expressions and to designate them by variable terms "S" and "P". Take, for example, the following sentences

(i) "Man is mortal"

(ii) "Alexander the Great is the son of the Macedonian king Philip."

In order to say that these two sentences have the same form (are "isomorphic") we must be able to correlate "Man" with "Alexander the Great" and "mortal" with "son of the Macedonian king Philip" in such a way that we can consider them as particular cases of some general term - here subject and predicate - which are connected by the logical constant "is".

The important thing is that there is no other way of establishing what the form of an entity is except in terms of "invariant under given transformations" or in other words, of "constant structure of some variable context". What follows from this is that a <u>system of reference</u> i.e. a set of entities similar in some respect to the entity in question <u>must be known</u>, otherwise the terms "invariant" and "transformation" or "constant" and "variable" are not uniquely determined. So, for example, the proposition "Alexander is the son of Philip". The form of this

For every proposition there is a set of propositional functions which can be taken as its form.

proposition can be established in a number of ways relative to the given system of reference, e.g.

(a) Alexander is the son of x

(b) x is the son of Philip

(c) x is the son of y

(d) Alexander x Philip

(e) x R Philip

(f) Alexander R x

(g) x R y

Which of these propositional functions is the form of the given proposition depends on which of its constituents are constant and which are variable in all propositions of the set under consideration.

Now two difficult questions arise when we want to find out the logical form of a class of sentences.

(1) In order to discover constants do we simply look for the verbal pattern of the given class? In other words, are we satisfied only to establish which verbal expressions repeatedly occur in certain positions, because this is the only observable constant thing in a sentence?

(2) Do we make correlations of the remaining expressions more or less arbitrarily? If not, what is it that does guide us?

As to the first question, the verbal pattern of a class of sentences in ordinary language is not the same thing as their

logical form. The most that can be said is that a verbal pattern corresponds to some degree to logical form and that in order to discover the latter, the former is a very useful practical guide.

However there are well known disagreements. Two sentences in two different languages may have the same logical form without having a single word in common. On the other hand two sentences in the same language may have different logical form in spite of having the same constants at the same relative position. A well known example is the distinction between "is" in the sense of class-membership and "is" in the sense of class-inclusion.

If there are such discrepancies between logical and verbal form, how can one know where they take place? Also, if the logical form is somehow disguised and cannot be established by simple observation of language signs - at least in the case of ordinary language - how is it to be discovered?

The first approach to an answer might be: We must take into account the meaning of the given sentences in order to make sure whether logical form is identical with their common verbal pattern. When we have the two following sentences:

"Julius Caesar was a Roman"

"Each Roman citizen was a free man"

we immediately know that the connective "was" does not express the

same relationship in both cases because we know what the words "Julius Caesar" and "roman citizen" mean, and we see that the subject in the first sentence is an individual and the subject in the second sentence is a class. However, something is obviously presupposed in this procedure, namely we take for granted that we already know how the meanings of descriptive words influence the logical meaning of the constant. Here the meanings of one type (when we have words which name individuals) made the constant "was" mean class-membership, and the meanings of another type (in the case of words which designate classes) made the constant "was" mean "class-inclusion".

Obviously we have explicit rules in some cases such as the one mentioned. How did we get them? In the same way as we learn many other things. At first we do not notice the necessary distinctions in the usage of some important word such as "is" in the given example. The consequence is confusion, which may be unnoticed for a long time. However, one day the difficulty is discovered and the method for the removal of the confusion is suggested, that is, a rule which makes explicit a previously unobserved distinction.

Thus failure to distinguish between the two meanings of the constant "is" led Ernst Schröder to inconsistencies in his "Vorlesungen über die Algebra der Logik". It was Frege who pointed out not only the confusion in Schröder's system arising

out of the identification of individuals with singular classes, but also suggested that these two meanings of the connective "is" should be clearly separated.[1]

Very often we have no explicit rules which determine the connection between the meaning of a constant and the type of context in which it is used in ordinary language. In view of the enormous complexity and richness of ordinary language it is hardly ever possible to compile a complete list of such rules of meaning for the logical constants. This is one more reason for constructing artificial languages. Here the logical constants are given fixed implicit meanings through axioms or through truth tables. These rules of meaning are also explicitly stated through semantic rules. Here the specific context cannot influence the meaning of constants. Therefore logical form does not deviate from the form of language.

To return to ordinary language, one might say that whenever we lack rules of the kind mentioned above, we must rely upon our intuition to determine the type of function which a logical constant plays in a given context.

Thus our procedure in looking for the constant constituents of the logical form of a class of sentences is roughly this:

(1) We try to see which are the identical words which

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1. Frege "A Critical Elucidation of some Points in E. Schröder's Vorlesungen über die Algebra der Logik" Translation from the Philosophical Writings of Gotlob Frege, ed. Geach and Black, Oxford, 1952.

occupy the same relative positions in all sentences.

(ii) We investigate the interrelations of the meanings of these identical words and the meanings of the other words in each sentence and make sure that in our communication they perform the same type of function in all cases (i.e. that they express the same content).

(iii) If there is a divergence between these two criteria we attach much more importance to the function than to the verbal identity and identity of position. Take for example the following sentences:

(1) "If a body falls freely it will accelerate at the rate of 9.81 meters per second."

(2) "Even if he is a little nervous he is still a good man"

(3) "When we warm a body, it expands."

We shall obviously decide that only (1) and (3) have the same logical form of a conditional sentence, although the function of conditional connective is played in one case by the word "if" and in the other by the word "when". (2) has not the same logical form as (1) because in the given context the word "if" has a concessive meaning synonymous with "although".

Now we come to our second question. How do we correlate the non-identical constituents of a class of sentences with each other? Obviously this procedure cannot be arbitrary. For instance, in the following two sentences:

(1) ~~Iron is a metal.~~

"Hyperons are atomic particles."

~~1935~~

Planck is the founder of the quantum theory

(2) ~~"Peter is John's father"~~ "Planck

hyperon

the correlation of ~~"neutral"~~ with ~~"John's father"~~ would be misleading, because the meanings of these two expressions belong to two different types. They cannot be used in a similar way, which implies that they perform different functions. This is easily seen when we try to make inferences of the same structure. E.g. In the case of immediate inference by conversion we get the following conclusions from (1) and (2):

(1') ~~"Some angels are dogs"~~ "Some atomic particles are hyperons."

(2') ~~"John's father is Peter"~~ "The founder of the quantum theory is Planck."

and these two are essentially different in structure.

So it seems that the conclusion follows: in both cases - in determining the constants and establishing one-to-one correlation between the remaining descriptive expressions so that they can be replaced by variables - we do essentially the same thing, namely, we study what the words mean and what type of meaning they belong to. In order to do that properly we must know either intuitively how the words are used in various contexts and what sort of function they perform, or in the most favourable case we should know the rules which determine explicitly the use of these words and the type of function which they perform.

In the artificial language we can construct these rules and leave nothing to intuition. However, in ordinary language we already have a usage of words as something given. We have some implicit rules which are obeyed intuitively. So the only thing

which can be done is to make these rules explicit, which is an almost impossible task in view of their great number and the ambiguity of many expressions.

By noticing the most general types of usage in ordinary language as a whole, we come to the well known categories of meaning such as: constants of individuals, predicates, relations and classes, variables of individuals, predicates, relations and classes, universal and existential operator, and logical connectives of implication, disjunction, negation, etc.

A conceptualist would express all this by saying that we distinguish between terms, sentences, etc. (verbal expressions) on the one hand, and extralinguistic mental entities expressed by terms (concepts, propositions, etc.) on the other. What we look for when we try to establish the logical form of a given class of statements is the invariant conceptual framework of propositions expressed by given sentences. To get this framework by eliminating concepts which vary from proposition to proposition, and substituting for them the logical categories to which they belong, and then connecting these categories by concepts which remained invariant through the whole sequence of propositions.

(8) Form of inference

So far we have discussed only problems arising in connection with the logical form of propositions. We have seen that Aristotle and all his followers in the next twenty-two centuries

hold the formula "S is P" to express the logical form of the only kind of statements they had investigated, i.e. attributive statements. Logicians in the nineteenth century, De Morgan and Lachelier, were the first to draw attention to the relational propositions. When Frege introduced the mathematical concept of function into logic it became possible to express the general form of all propositions by a propositional function "F(x...)", supplemented by the universal or existential operator.

When we now come to the problem of the form of inferences we have to apply the same procedure as in the case of propositions.

First we must determine the system of reference, i.e. the class of inferences which the form we want to establish is relative to.

Then we find identical constituents of all particular inferences (identical from the point of view of their function, i.e. of the category of meaning to which they belong, not from the point of view of their verbal expression or only their relative position). We bring into one-to-one correlation the non-identical expressions and substitute variables for them. In such a way we get the common skeleton of the inferential process which has taken place in all particular inferences. Such skeletons are Aristotle's figures of syllogism or a number of schemata of relational inferences discovered in symbolic logic in the last hundred years. It has been shown that in some cases we do not

need to exhibit the structure of particular propositions and that the logical form of the inference as a whole may be sufficiently clearly represented by substituting propositional variables and constants for whole sentences. In such a way we have got an additional general category of meaning - the proposition.

In the case of the logical form of inference as opposed to that of propositions we are not satisfied just to have a schema of a particular class of actual inferences. We want also to know on what grounds the conclusion follows from its premises. Inferences, which are the subject of logic, are characterised by the claim that the conclusion is true - provided that the premises are true. Whenever we make this claim and really mean it, we must have had some criteria of choice between various possible derivations, and these criteria are constituted by sets of rules specific or general, which might be either intuitively guessed or known in a rational way. All these rules express our knowledge that from the truth of premises the truth of conclusion follows.

The form of a set of inferences is completely shown only when these rules have been explicitly stated. Here a point of great importance and with far-reaching consequences should be raised.

If the meaning of form is uniquely determined only relatively to a system of references, then it follows that in

relation to various systems of reference we have also various overlapping sets of rules rather than a unique one. There are some general rules underlying all sets and some specific ones varying from set to set. This can be easily seen from the following example:

Inference I

"If the temperature of a body is raised it will expand.

The temperature of this stove is raised.

Therefore it will expand."

Inference II

"If the temperature of water is raised to 100°C it will boil.

The temperature of water in this vessel has been raised to 100°C.

Therefore it is boiling."

If we now ask ourselves what our knowledge that the truth of premises implies the truth of the conclusion consists in, we shall firstly notice a general rule implied in both cases. It may be expressed in this way: "If it is true that raising the temperature of a body implies certain phenomena x, then whenever it is true that the temperature of a body has been raised, it is true to say that the phenomena x has occurred". We deliberately do not give the expression of this rule a more general form than necessary. As it is supposed that our system

of reference is this set of only two inferences, "raising of temperature" is a constant and therefore a formal feature.

Besides this general rule there are some special ones in both cases. In case I we have the rule: "Raising the temperature of a body implies its expanding for all the values of variable 'body' except for water in the temperature-interval 0 - 4°C."

In case II we have the rule: "Raising the temperature of water to 100°C implies its boiling under the air pressure of 1 atmosphere and all the other ordinary conditions on the earth."

These special rules obviously qualify the range of application of general rules. If we fail to take into account the limits of the application of a general rule and conditions under which it holds, we shall be misled in some cases. Namely, we shall draw false conclusions and think that they are true, being derived from true premises in accordance with the rules.

We can express the role of these special rules in another way by saying that they specify the conditions under which the premises are true. We have seen that the truth of an empirical proposition depends on the given context of application etc. Therefore we must have special rules adjusted to the specific case in question.

Finally, there is also a third way of expressing the same idea. We may say that in the sentences which formulate the rules of our two inferences some variable appear. As these rules apply to a limited field they will hold only for some values of the variables. The role of special rules is then to limit the range of values for variables in the general rules in such a way that whenever these rules are applied for the derivation of some conclusion from the given premises, we always get the true conclusion from the true premises.

For example in (I) the special rule specifies the range of values of the variable "body" by excluding water in the temperature-interval 0° - 4°C. In (II) the special rule limits the range of the values of the variable "water" by saying "water under air-pressure of 1 atmosphere etc.".

After these explanations, an answer may be given to the question: how do we determine the rules which govern the derivation of conclusions in a set of inferences. The answer is:

(1) We make a set of rules for each particular inference trying to state explicitly in the most economical way our reasons for believing that from the truth of the premises the truth of the conclusion will follow.

(2) We notice the constant features in all these sets of rules on the one hand, and the variable features on the other. We

construct general rules by making one-to-one correlation of variable expressions, substituting symbols of variables for them, and connecting them by constants. We also construct special rules by determining the range of variables for which the general rule will hold (will be a truth-condition) in the given particular inference.

(3) Then we keep general rules (general in relation to the given system of reference S) and consider them as constituents of the logical form of the system of inferences S. As to the special rules, we eliminate them while theoretically trying to find out the general logical form of S. However we take them again into account when we apply these formal general rules to the specific case in question.

Thus the form of an inference relative to a given set of inferences is shown when (i) the form of constituent propositions in all the members of the set is exhibited; (ii) the general rules which govern the derivation of all conclusions in the set are explicitly stated.

Form of theories and systems

To know the form of a theory or system belonging to a particular science is firstly to know the form of all inferences contained in the theory. In most cases there is not a single unique form of all of them. Then we shall distinguish various

groups with constant features and consequently we shall get several form of inferences. For example

(a)	(b)	(c)
If P then Q	P or Q	P
P	Not P	Q
Therefore Q	Therefore Q	Therefore P and Q

etc.

Secondly a theory as a whole has always certain criteria according to which some statements make sense and other do not within the framework of the theory. In other words, the language in which the theory is expressed contains criteria for distinguishing meaningful from meaningless statements. These criteria are held implicitly before a formalisation of the theory is attempted. In most cases in empirical sciences they are rather vauge. For example, it would be considered meaningless by all microphysicians to say: "Electrons are able to love each other". However, some of them (Bohr, Jordan) think that in view of Heisenberg's principle of indeterminacy it makes sense to say: "Electrons have free will".

This lack of generally accepted criteria of meaningfulness in some borderline cases prevents us from stating quite precisely "the rules of formation" in the language of the given theory. There remain some areas of vagueness. However these areas of vagueness can also be investigated and limited.[1] At any rate

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1. Max Black, Vagueness, Philosophy of Science, v. 4 (1937), pp. 427-455.

if a theory T can be formalised at all (in other words if its form can be made explicit), its full formalisation comprises the explicit stating of "rules of formation" i.e. criteria of which statements are meaningful in the language of T.

Thirdly, a theory as a whole has some general rules of inference which are truth-conditions for all inferences which belong to it.

Fourthly, the terms which cannot be defined within the framework of the theory itself, but which are used for the definition of all other terms of the theory, must be enumerated. It is clear that there must be some terms whose meaning is presupposed as intuitively clear or that have been defined in some higher-level theory. Otherwise we should have to admit that there is a vicious circle in the theory and that the relation "definiendum - definiens" is used as a symmetrical relation which leads us into confusion.

The enumeration of the most simple undefined concepts and the definition of all the other more complex ones in their terms gives us an insight into the formal structure of the meanings of all terms used in the given theory.

Finally, the form of a theory is fully revealed only when all its assumptions are explicitly stated. Without this condition being satisfied, the order of inferences would not be sufficiently clear.

The premises of inferences must be true statements. If this is so, then if there is not to be a vicious circle in the theory (the same statement A whose truth is proved by derivation from the statement B being used to prove B itself) there must be premises which are not proved to be true, but which are used for proving the truth of all the rest. Thus we say that a theory is completely formalised only when all five conditions have been satisfied. We can put them in a better order by indicating progressive steps in the process of constructing the theory in its formalised version:

(a) enumeration of undefined terms

(b) the explicit statement of the rules of formation of meaningful propositions

(c) The explicit statement of the initial assumptions (axioms)

(d) the explicit statement of the rules of inference

(e) The restatement of the inferences themselves in a formal way.

We call a theory incompletely formalised when either only a part of these conditions is fulfilled, or at least one of them is fulfilled only partially, leaving out some hidden implicit assumptions or rules of formation and inference, etc. So, for instance, Aristotle formalised the theory of syllogism to a very great extent, but not completely. For example, he failed to distinguish between "is" as class-membership, and "is" as class-inclusion.

6. The relation between form and content. The concept of logical form

We have seen that whenever we have found ourselves confronted with the task of determining the logical form of a certain unit (proposition, inference, theory) we have had to take into account a certain class of similar units as our system of reference and then to proceed by investigating its constant and variable features.

Many logicians seem to be unaware of this relativity of the concept "form" and more particularly "logical form". The Kantian way of saying that logic has to investigate "pure forms" quite independently of their "content" is still very popular. However, this way of speaking presupposes certain assumptions which are either false or very obscure and meaningless - in the sense that they cannot be either verified or falsified.

If we speak of "pure forms" this makes perfectly good sense within the framework of Kant's doctrine. Kant presupposes that there is a sharp distinction between experience and understanding. There are descriptive data about "things in themselves" which are acquired through our senses. On the other hand there are fixed "a priori" concepts constituents of reason. Experiential descriptive data constitute the content of thought; they are unordered, completely chaotic. The function of "a priori" concepts is to bring order and to organise experience. They are "forms"

of thought. They are pure in the sense that they are known independently of (although not necessarily before) any experience, and can not be affected by it.

Thus an absolute difference between form and content is established. There might have been good reasons for holding such a view in a time when there was a general belief in absolute time and absolute space (whose forms were presumably exposed in Euclidian geometry) and when Aristotelian logic had been considered for centuries to be the only possible logic. However, things have changed very much since those days. The discovery of non-euclidian geometry was a deadly blow for the Kantian conception. Instead of a unique system of spacial forms we got several opposite systems, each of which was self-consistent and able to do the work of organisation of our sensory experience. There might be reasons for deciding that one of them was preferable but they would be mainly pragmatic. Another blow was struck by the theory of relativity, which destroyed the conception of absolute physical space and time.

A similar process took place in logic. Many new fundamental concepts like "conjunction", "function", "variable" etc. had to be incorporated into formal logic and some of Kant's pure forms of reason had to be discarded. This suggests in itself that there is no closed set of absolute forms of reason

However, a Hegelian might contend that the progress of our logic (subjective logic, he would say) does not imply that there is not a closed absolute system of logical forms (objective logic, he would say). And he might assert: in fact there is such a system. This system is the all-pervasive structure of the whole of reality which is absolute in itself, but is becoming known to men only step by step.

This contention either does not help us very much or is misleading. It does not help us because, apart from the lack of any evidence or any other reason whatever for holding that this supposed structure of reality is absolute, it shifts the meaning of the term "logical form" and introduces ontological problems which we are not concerned with at all. What we want to know is only: is there any sense in holding that there are absolute or "pure" forms of thought which are related in an exclusive way to the content of thought?

On the other hand, this Hegelian contention and any other doctrine which might attempt to explain the development of logic as a progressive revelation of some hidden absolute logical structure, so that at any given moment there is a logical system which represents the best possible approximation to the hidden idea, fails to account for the fact that we know nowadays various systems of logical forms, some of them being mutually incompatible, and nevertheless, each of them is equally

legitimate and each of them can be successfully applied in science. As a consequence, saying that a concept or principle is a logical form always means that it is so in a given logical system and not in any absolute way.

For example, "the principle of excluded middle" is a logical form in Principia Mathematica and similar systems. In Brouwer's and Lukasiewicz's three-valued logic it is not a logical form, but an empirical principle which is true only under certain conditions (for finite sets in Brouwer, for value 1 (not for value $\frac{1}{2}$) of p in the formula "$p \vee \sim p$" in Lukasiewicz.

If all this is so, the consequence seems to be that the difference between logical form and content is also relative to the given system, and that what is form in one system can be content in another and vice versa.

What is meant by this can easily be seen from the following examples. We take sentences from four languages of different levels. (I) is physical language; (II) is the general language of the philosophy of science ~~(ontology)~~; (III) is the language of logic; (IV) is the language of meta-logic.

I. (a) The expanding of a gas is the cause of the decrease of its pressure.
   (b) The heating of water is the cause of its evaporating.
   (c) Gravitation is the cause of free fall.

II. (a) The concept of cause implies the concept of effect.
   (b) The notion of the future implies the notion of the present.

(c) The concept of a goal implies the concept of a living organism striving to reach it.

III. (a) "Not p" is alternative denial of p and p.

(b) "P implies Q" is alternative denial of P and not Q

(c) "P is in conjunction with Q" is alternative denial of not P and Q.

IV. (a) "Alternative denial" is a constant logical relation between two expressions in a sentence such that the sentence is true if and only if either both expressions are false, or if one is true and the other false.

(b) "Implication" is a constant logical relation between two expressions in a sentence such that the sentence is true if and only if the first expression is false or the other is true.

(c) "Conjunction" is a constant logical relation between two expressions in a sentence such that the sentence is true if and only if both expressions are true.

Now the form of the given sentences in the language (I) is "x is the cause of y". The content is constituted by the values of the variables x and y.

In the language (II) the term "cause" is only one of the values of a variable. In this language it has not the status of a formal constituent any more. The form of the given sentences in (II) is "t implies v".

The language (III) is the language of a particular system of logic in which the only constant is Sheffer's "alternative denial" ("/"). All the other constants can completely disappear from the language after they have been defined in terms of alternative denial. Thus they cease to be formal constituents of the system. The form of the given sentences in (III) is "A is alternative denial of B".

Finally - what was a formal constituent in the object language of logic - the term "alternative denial" becomes the constituent of the content spoken about in the language of meta-logic. The formal concepts in this higher-level language are concepts like "constant logical relation", "expression", "sentence", "true", etc.

The conclusion is that, <u>linguistically the relation between form and content appears to be the relation between constants and variables in a given language.</u> As we remember well, here we do not mean by constants and variables simply constant and variable <u>verbal</u> expressions, but expressions used to designate constants and variables. If we allow here the term "concept" into our terminology, we may express the same idea by saying that <u>the relation between form and content appears to be the relation between constants and variables in a given conceptual framework.</u>

There are also other ways of saying the same thing. For example, we can substitute the terms "determinate" (det) or "definite"

for "constant" and the term "indeterminate" or "indefinite" for "variable". This terminology was often used in philosophy. Form was usually identified with what is fixed, limited, determined, and content with what is flexible, flowing, indeterminate, changing from case to case, and from one moment to another. The formal logicians sometimes express variables by blank space (something indetermined) instead of latin letters.

However, a powerful objection to this relativisation of form and content might come from those linguistic philosophers who wish to make a sharper distinction between form and content and to save the notion of "a-priority" for at least some logical concepts. They may give the following argument:

The logical forms are the forms of a given language. They cannot be affected in any way by what we speak about, i.e. by any experiential content, because we have chosen them by convention, and it is by convention that we have made them mean what they mean. E.g. the meaning of the implication connective "$\supset$" cannot be affected by the meaning of the connected expressions $\underline{P}$ and $\underline{Q}$. The sign "$\supset$" simply means that "$\underline{P} \supset \underline{Q}$" is true whenever $\underline{P}$ is false or $\underline{Q}$ is true. Also we lay down by our rules of formation and transformation that "$\sim(\underline{P} . \sim \underline{P})$" must be true no matter what P means - therefore it is a formal and at the same time an "a priori" constituent of the system. "A priori" here means "given by convention", and just because of that independent of any experiential content.

These explanations entail that although the difference between any particular logical form and content is relative to a given language, the difference between logical form and content within the framework of any given language is absolute in the sense that the former refers to linguistic conventions and the latter to knowledge obtained by experience.

This conception of "a priori" is a great improvement on Kant. Instead of committing oneself to asserting the existence of a closed and unique set of absolute concepts of reason, it implies only our freedom in choosing verbal conventions and constructing various languages. And this freedom is incontestable. The fact is that we can construct arbitrarily as many artificial languages as we like, and we can choose formal constituents and properties as we like.

However what is to be contested is that all these possible arbitrary linguistic conventions are logical forms. From our previous discussion of the problems of meaning and truth and from our definition of logic it follows:

(1) The class of logical forms is only a part of the class of arbitrary linguistic conventions.

(2) Logical forms are forms (constituents of structure) of a (meta) language in which the truth-conditions for some other (object) language are expressed.

(3) Being constituents of the formulae which express the truth-conditions for some theory, the logical forms cannot be

independent of the content of the theory. We choose such and only such linguistic devices which, at least in principle, can perform the function of attaining O-truth. When they are not able to function in that way we abandon them in order not to be misled.

(4) Therefore, the distinction between the logical form and content is relative in two senses:

(a) it is relative to a given language (i.e. logical theory or system which is expressed in that language);

(b) it is also relative in the sense that there is no cleavage, but a fundamental interdependence between logical form and content.

(1) follows from the definition of logical truth. Logical forms are those linguistic conventions which among other things are O-interpreted and O-applicable.

(2) follows from the definition of logic. When once the criteria of what logic is are established, only those forms which belong to a theory satisfying these criteria can be considered as logical forms.

(3) and (4). If logical forms are constituents of the schemata which express the truth-conditions of some theory, and if we take for granted that logical formulae have the characteristics of leading to empirical truth for <u>all</u> values of their variables, then there is a functional relationship between

logic and empirical knowledge, which can be expressed in the following way:

(a) All logical formulae of a logic must be transformable (it must in principle be possible to transform them/ into universal empirical true propositions of the theory to which it is correlated as its meta-theory. And vice versa all universal empirically true propositions of a theory T can be transformed into formulae of a logical meta-theory for T.

(b) If it turns out that such a transformability of a formula believed to be logical is not possible i.e. if it turns out that for at least one value of its variable within it does not lead us to empirically true propositions, we are bound to consider it to be no longer a logical formula.

Besides this distinction between logical forms and any linguistic conventions of an arbitrarily constructed calculus, another distinction between <u>forms of thought in general</u> and <u>logical forms</u> must be made.

We have seen that we can take any arbitrary set of propositions, inferences or theories for our system of reference and then find out what is the form of a particular proposition, inference or theory relative to it. We can establish in that way the form of meaningless or false propositions, wrong inferences, misleading theories, etc. It is not inconceivable that someone might use the term "logic" in such a broad sense as to cover all

these cases. Then the distinction between logical form and form of thought in general would disappear, and we should have to consider as logic not only any formalised scientific theory but also the formalisation of any thought or speech whatever. Sometimes we even go so far as to speak about the logic of a particular neurotic, or "the queer logic" of some argument based on purely emotive grounds. In all such cases we identify logic with the formal structure of any process of thought and speech. However, we are not so liberal (and vague) when we use the predicate "logical". We should never ascribe it to an inference or theory where the premises are true and the conclusions are false. For any inference or theory to be logical means that if the premises were true, no matter what they expressed, and whether they actually are true or false, the conclusions would also be necessarily true. That means that the class of logical forms is the sub-class of the class of all forms of thought. Only those structures of variables connected by constants will be considered as logical forms whose system of reference is not any arbitrary collection of expressions but a theory which satisfies the definition of logic. That again means:

(1) There is a theory T_1 expressed in an object-language L_1.
(2) There is a theory T_2 expressed in the meta-language of L_1 i.e. in L_2.

(3) T_2 lays down the O-truth conditions of T_1 i.e. it is a logical theory.

(4) Logical forms are constant and variable terms, rules of formation, transformation and meaning and all theorems (proved propositions) of T_2.

4. Abstract and concrete logic

We have seen that for each scientific theory its meta-theory, i.e. its logic, can be constructed. The degree of abstractness of logic will depend on the level of generality of the corresponding theory. The logic must not be weak to the extent that even empirically false statements can be logically proved. On the other hand the logic must not be strong to the extent that even empirically true statements of the given theory fall outside its narrow field of application. In other words, the logic L_j for the theory T_j must neither justify the cases which are excluded by T_j nor fail to give truth-conditions for cases which definitely belong to T_j.

Now if T_j is a special case of the more general theory T_k, the logical system corresponding to T_k that is L_k, will be a weaker and more abstract system. That means, it will exclude either some axioms or some rules of L_j or both (which entails that also some undefined terms can be excluded).

On the other hand, if T_j is a generalisation of some more special theory T_i, the logical system corresponding to

T_i, that is L_i, will be a stronger and more concrete system. It will contain some additional axioms or rules, which entails that also some specific undefined terms may be added. In this way we get a richer logic which will provide schemata for establishing the truth of specific propositions of T_i (which could not otherwise be shown to be true within the framework of L_j). However, the field of application of this richer logic is more restricted; outside T_i some of their axioms and rules would be redundant.

We can take as an example of an abstract logic the ordinary logic of Principia Mathematica and compare with it any special logic such as, for example, Woodger's logic of the biological theory of cell.[1]

The specific terms introduced by Woodger are the descriptive constants "P", "T", and "cell", where "P" denotes the relation part of "T" the relation before in time and "cell" the class of cells.

Another specific feature of Woodger's logic (theory T as he calls it) is the introduction of a number of postulates, some of them having the character of definitions of some further necessary terms, and the others playing the role of special axioms. All these specific terms, presupposed by Woodger either as

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1. J.H. Woodger, The Technique of Theory Construction, International Encyclopedia of Unified Science, vol.II,no.5. Chicago 1939.

undefined or defined by postulation, are characteristic for a natural science and cannot be met in any general logic like "PM". Some of them hold for all natural sciences as terms related to space and time: "part of", "momentary", "before in time", "coincides in time", "time-slice", "beginning slice", "end-slice". Some of them are purely biological, such as "cell", "to be immediately derived by division", "to arise immediately through fusion".

Woodger's special axioms (postulates which do not serve for definition) determin firstly some obvious general truths about spatial and temporal properties of things[1]. (e.g. "part of" and "before in time" are transitive relations) and secondly, some generally established truths about cells such as:

5.11 "If a thing x is a cell, then there are parts of x which stand in T (precede in time) to x.

5.12. "If a thing x is a cell, then there are parts of x to which x stands in T."

5.13. "No cell is a momentary thing."

5.14. "If two distinct cells have a part in common, then the first time-slice of one is a proper part of the last time-slice of the other, as the last time-slice of one is a proper part of the first time-slice of the other."[2].

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1. op.cit. sections 1 and 2, pp.33-35.

2. op.cit. section 5, p.40.

The last of these propositions is specifically biological. The first three can be satisfied by all things which are not momentary and have a beginning and an end in time.

Now Woodger showed how such a logical system can be used in order to prove logically a number of propositions about various properties of cells, the most important from the point of view of the theory of cells being the following four theorems in which the properties of division and fusion of cells are described:

5.52. "The relation in which one cell stands to another when the latter arises immediately by fusion from the former is a many-one and asymmetrical relation".

5.71. "No cell both divides and fuses with another cell."

5.72. "No cell arises both by division and by fusion."[1]

It would not be possible to prove in a rigid logical way these propositions by means of ordinary general logic of P.M.

However, not only do we need to construct similar special logical systems in order to get richer theories of proof, but also in order to avoid possible mistakes which might arise out of the application of a too abstract logic to a concrete field.

Neumann and Birkhoff showed that the principle of distributivity is not applicable to quantum mechanics. This

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1. op.cit. pp.56-58.

entails that if we use it as a premise of an inference where the symbols are interpreted as the concepts of quantum mechanics, we shall be misled and we shall get false statements, believing that they are true.

This example illustrates well the dangers which one may meet whenever one applies general formal logic without the necessary adjustments. These adjustments may consist in the elimination of some axioms and rules or (what is much more often the case) in the incorporation of some additional postulates and rules which express the specific features of the given field of applications.

We have already seen, while discussing the logical form of inference that the set of rules which justify a conclusion might often include besides the general rules some special ones. And we have seen how we may be mistaken when we draw conclusions which express some empirical propositions only in accordance with some very general rules, neglecting the specific context in which our premises have their full meaning. This is so because our premises, being empirical statements, are relative to a certain context (to a given time-interval, spatial system, and various other conditions). We need not express all the necessary qualifications in the statement itself, and we almost never do that. However, we make an effort to bear them in mind, and where a situation arises where they become important we use them in a somewhat intuitive way. Therefore if we warm a piece of ice we

shall not expect the volume of water to expand although it is true, generally speaking, that things expand when heated. And also if we are at the North Pole we shall not expect water to boil at 100°C although we always do so in ordinary conditions, etc. In all these cases we compensate for the incompleteness of our premises by using our intuition. Superficially one might say that in these cases we break our rules. In fact we only restrict the field of their application. We specify the premises we ordinarily use, and then find out that nothing follows from them concerning our relevant case (or that false conclusions follow). Then we look for some other premises.

This is roughly how our thought works in ordinary life. We use then all sorts of non-formal procedures.

Now when we want to think or write in a logically rigorous way we must explicitly state as many as possible of these intuitive considerations which help us to avoid false and paradoxical conclusions.

In this way we construct a more concrete pattern of thought, although it is rigorous in the same way in which the abstract one is. The ideal would be to satisfy both the demands of rigour and of concreteness; but obviously this can not be done once and for ever for all cases. Each particular situation and each concrete problem we try to solve are unique.

On the other hand logical rules are and will always remain general - on different levels of generality.

To apply a general rule is to follow an abstract scheme of reasoning, neglecting necessarily some specific features of the concrete situation. Therefore there is no guarantee that at the end of the operation we shall get an empirically true sentence. Our rule is only a "condition sine qua non". Although we shall very probably[1] be wrong if we do not think in accordance with a certain general logical scheme of reasoning, on the other hand, we might do so and still be wrong. Some other condition must also be fulfilled and at least some of them should be formulated by the more special rules.

Of course, all this speaking about the necessary concreteness of our thought and the necessity of constructing special logical theories makes sense only when the problem of the application of logic arises. But the problem of application is there, although sometimes for the sake of simplification we must ignore it.

We need logic not in order to play a game, but in order to be more successful in solving our problems, in science, philosophy and ordinary life.

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1. Not certainly, because it might happen that we get from false premises, using false rules of inference, a true conclusion by pure chance.

The conclusions, then which follow from the conception of logic advocated here seem to be:

1. Logic stands in a double relationship to science. On the one hand, the starting point for the formulation of its rules are those intuitive procedures of human thought which have always in the past brought us to reliable and practically verified results. On the other hand, on this basis a superstructure of principles and rules is constructed such as to be applicable (at least indirectly) for obtaining objectively true propositions in a certain field.

2. If there is such a functional relationship between logic and science, the construction of symbolic structures should be supplemented by the analysis of the conditions of their interpretation and application. (In other words, the work on the problems of the formal theory of proof should be balanced by the work on the theory of meaning and methodology of science.).

3. If logic is essentially a science about the conditions of attaining objective truth, it follows, then, that one of the tendencies of the further development of logic should be the construction of more concrete, special logical systems, adjusted to the peculiar features of the fields to which they might be applicable.