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FOURIER ANALYSIS OF ROTATIONALLY BROADENED STELLAR SPECTRA

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Abstract. A complex combination of physical processes affecting mainly the *shape* of a spectral line formed in stellar atmosphere can be better distinguished at low Fourier frequiencies than in the profile itself. Considering the analysis of stellar rotationally broadened line as a typical example of those astrophysical problems where it is more advantageous to use the Fourier transform of the profile, we present a method for determination of stellar projected rotational velocity and limb-darkening coefficient.

1. INTRODUCTION

In the past the use of the Fourier transform for spectral line shape analysis has centered around several active areas as microturbulence and macroturbulence in stellar atmospheres, stellar rotation, the detection of Zeeman splitting and global velocity fields in stellar atmospheres (Smith and Gray, 1976).

Many types of line broadening functions involving different kinds of processes, especially those occuring on a macroscopic scale due to collective motions, are quite different from one another. These differences may be greately smoothed when convolution is performed. A theoretical profile may appear to reproduce an observed profile fairly well, when in fact, there are small but systematic differences extending over the profile. Often only subtle variations in the shape of the core and far wings of the broadened profile can give clues to the mechanism. The power of the Fourier technique rests on the fact that subtle but systematic variations of the line profile show up as significant and easily identifiable signatures in the amplitudes of certain Fourier components. The Fourier analysis provides a sensitive way to use all of the information contained in the shape of a line profile to estimate the contributions of various types of velocity fields.

The Fourier transform analysis is most effective when broadening mechanisms in the stellar atmosphere affect the detailed *shape* of the observed flux profile. However it is not such a powerful tool for analysis of physical processes related to *global* parameters of the profile as it is, for example, the relation of chemical composition to the equivalent width of the profile.

2. ROTATIONAL BROADENING

Rotation is the dominate line broadening mechanism in a large number of stars. Let us consider the star rotating with the equatorial velocity V_e, inclined by the angle i. Under the condition that the shape of intrinsic spectrum does not depend significantly on the position on a stellar surface, the observed normalized profile is given by the relation (Gray, 1976) :

$$R(t) = \int_{-1}^{1} H(t-y)G(y)\mathrm{d}y, \tag{1}$$

where $H(\lambda)$ is the normalized intrinsic spectrum as it would be observed on the apparent disc of the star,

$$y = \frac{\lambda - \lambda_0}{\Delta\lambda_D} \quad ; \quad \Delta\lambda_D = \frac{\lambda_0}{c} V_e \sin i,$$

and the rotational profile $G(y)$ is defined by

$$G(y) = \frac{\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} I_c'(M,\lambda_0)\mathrm{d}z}{\int_{-1}^{1}\mathrm{d}y \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} I_c'(M,\lambda_0)\mathrm{d}z},$$

with the axis y, chosen to be oriented in the direction of the spectrum dispersion and axis z chosen to be oriented in the direction of the projection of stellar rotational axis on to the tangent plane. The emergent intensity in the continuum $I_c'(M,\lambda_0)$ corresponds to the area defined by angle Θ between the line of sight and the outward normal at the point M.

By assuming a limb darkening law for the continuum intensity distribution on the apparent disc of the star in form of :

$$I_c'(M,\lambda_0) = I_c(\lambda_0)[1 - \varepsilon + \varepsilon\cos\Theta],$$

for the star with uniform surface intensity distribution $I_c(\lambda_0) = \text{const.}$, the observed spectrum of the rotating star can be represented as a convolution of the intrinsic spectrum of a nonrotating star and the rotational profile defined as :

$$G(y) = \begin{cases} \frac{2(1-\varepsilon)}{\pi(1-\varepsilon/3)}\sqrt{1-y^2} + \frac{\varepsilon}{2(1-\varepsilon/3)}(1-y^2) & y < 1 \\ 0 & y \geq 1 \end{cases}, \tag{2}$$

where limb darkening coefficient ε, as a slowly varying function of λ is considered constant over the line profile.

3. FOURIER ANALYSIS

The mathematical treatment of the problem can be simplified by introducing for a given distribution $F(\lambda)$ of the space coordinate λ its Fourier transform $f(\omega)$ of spatial frequency (radians/length) ω defined as :

$$f(\omega) = N^{-1} \sum_{\lambda} F(\lambda) \exp\left(-\imath\omega\lambda \frac{2\pi}{N}\right)$$

where $\imath$ denotes the imaginary unity and N is the number of measurements.

For the Fourier transform pairs $R(\lambda), r(\omega)$; $H(\lambda), h(\omega)$ and $G(\lambda), g(\omega)$ the relation (1) can be expressed in the Fourier domain :

$$r(\omega) = h(\omega) \cdot g(\omega)$$

This conversion of convolutions of functions in the wavelength domain to their products in the Fourier domain is the most obvious reason for the choice of Fourier domain for analysis of stellar spectra, since in many cases the broadening of a spectral line can be adequately represented by some unbroadened intrinsic profile convolved with a function which depends on a geometry of the motions involved.

Further, the Fourier transform of an absolutely integrable function is known to tend to zero as $\omega \rightarrow \infty$ and the smoother the function $R(\lambda)$ the faster its Fourier transform falls. The convolution itself acts as a low-pass filter that eliminates all frequencies beyond some truncation frequency. It means that the essential information about the shape of the spectral line is described by the components at lower frequencies. In contrast, noise is distributed over all Fourier frequencies providing that the low frequency Fourier components are less affected by the noise than data in original domain.

As a consequence, many physical functions have characteristic signatures in their transforms showing up as rather large amplitude differences in Fourier domain comparing with shape differences that are often difficult to detect in original domain.

The finite extension of the rotational profile (2) implies that $\mathrm{g}(\omega)$ has zero amplitude at certain Fourier frequencies ω_i. Table I list the positions of the first (s_1), second (s_2) and third (s_3) zero of the rotational profile, where $\mathrm{s}_i = \Delta\lambda_{\mathrm{D}}\omega_i$. Important property is that the multiplication of $\mathrm{g}(\omega)$ by the transform of $\mathrm{H}(\lambda)$ will not change the positions of zeros s_1, s_2, ..., so the method is applicable for all stars where the dominant broadening mechanisms act as a convolution with the rotational velocity field.

Since $G(\lambda)$ scales with $\mathrm{V_e} \sin i$ (through y) and ε, it is assumed that only two parameters need to be measured. In fact one needs to determine only the positions of two zeros of the Fourier transform of the roational profile in order to determine the projected rotational velocity and limb-darkening coefficient. However the position of the third zero should be also used to estimate the error bars on two parameters, providing the information if the real star under investigation, conforms to our model. This is true for all broadening mechamisms at microscopic level, microturbulence and the isotropic macroturbulence that is usually assumed to take the form of convolution with a Gaussian broadening function.

TABLE I
Zeros of Fourier amplitude of rotation profile in function of limb darkening coefficient

ε	s_1	s_2	s_3
0.0	0.609	1.116	1.618
0.1	0.616	1.121	1.623
0.2	0.623	1.127	1.628
0.3	0.631	1.134	1.634
0.4	0.640	1.142	1.642
0.5	0.649	1.151	1.650
0.6	0.660	1.162	1.661
0.7	0.672	1.175	1.674
0.8	0.685	1.190	1.690
0.9	0.700	1.208	1.711
1.0	0.716	1.230	1.736

Note that the microturbulence can produce a sidelobe structure as well as a characteristic structure of rotation, however the additional zeros of the Fourier transform introduced by microturbulence does not affect the position of zeros of the rotational profile.

Concerning the accuracy of resulting estimates for $V_e \sin i$ and ε, in addition to limb angle dependence of line profile and departures from linear model, one should also be concerned with the existence of global velocity fields. The relevance of these problems to rotating line profiles has been investigated in Jankov, Unruh, Collier-Cameron (1995).

Center-to-limb variation of the intrinsic profile has been analysed calculating specific intensity profiles for different limb angles on the stellar disc. Figure 1. shows the resulting profiles with and without limb angle dependence of the intinsic profile and resulting estimates for a star with $V_e \sin i = 91 \mathrm{kms}^{-1}$ and $\varepsilon = 0.65$.

In spite of the fact that the deviation of the observed profile from the calculated profile are rather small, and that the estimate $V_e \sin i = 90.9 \mathrm{kms}^{-1}$ is correct the determination of ε (=0.75) is overestimated by neglecting these second order effects.

As other example we use a more general formulation of the macroturbulence including an arbitrary velocity distribution that, in general, cannot be represented by a convolution. Most of all stars have a macroturbulent velocity distribution which is not the often assumed isotropic Gaussian. The macroturbulence allowed to move only in radial and tangential streams (i.e. anisotropically) with a Gaussian distribution of velocity amplitudes is often called the radial-tangential macroturbulence.

Figure 2. shows the profile calculated performing disk integrations and using the model of radial-tangential macroturbulence. This effect do not seem to have a major influence on the profiles in the wavelength domain, however it can be clearly seen in the Fourier domain. One can notice overestimated value of $V_e \sin i = 92.4 \mathrm{kms}^{-1}$ but also a significant interaction between the two parameters (velocity field and limb-darkening coefficient) leading to overestimated $\varepsilon(= 0.73)$.

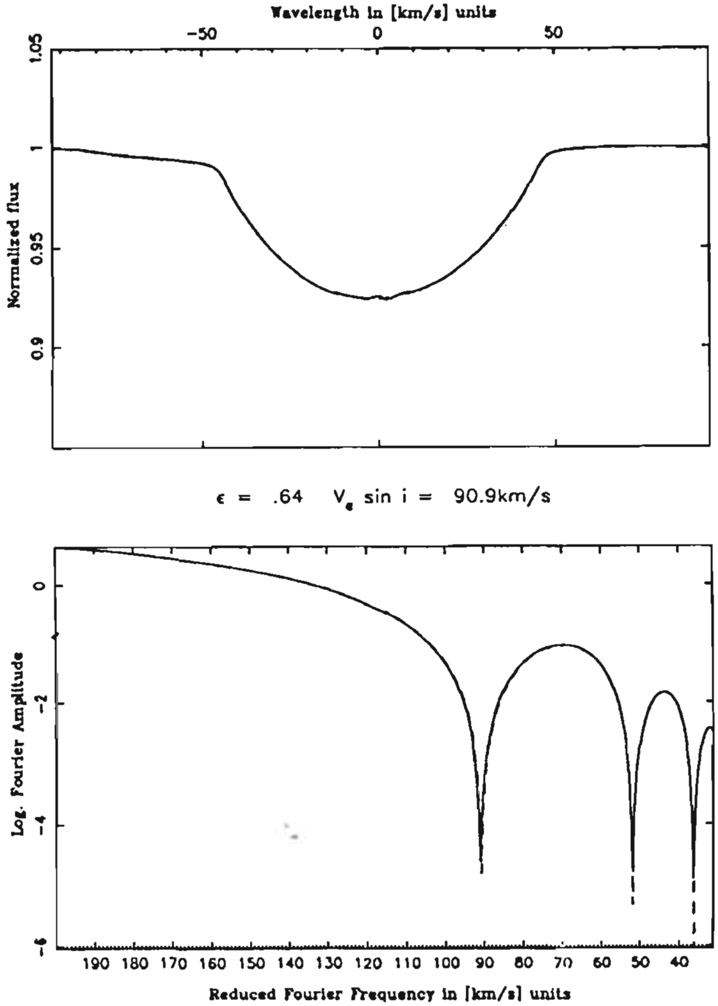

Fig. 1. **Top.** Full line represents the profile calculated using the equations (1) and (2) with $V_e \sin i = 91 \mathrm{kms}^{-1}$ and $\varepsilon = 0.65$, under the presumption that the shape of intrinsic spectrum does not depend on the position on a stellar surface. Dashed line represents the profile calculated performing disk integrations where a model photosphere for a K0 IV star is used to generate $H(\lambda)$. **Bottom.** Fourier transforms of corresponding profiles are shown. Note that the abscisa is reduced to velocity kms^{-1} units.

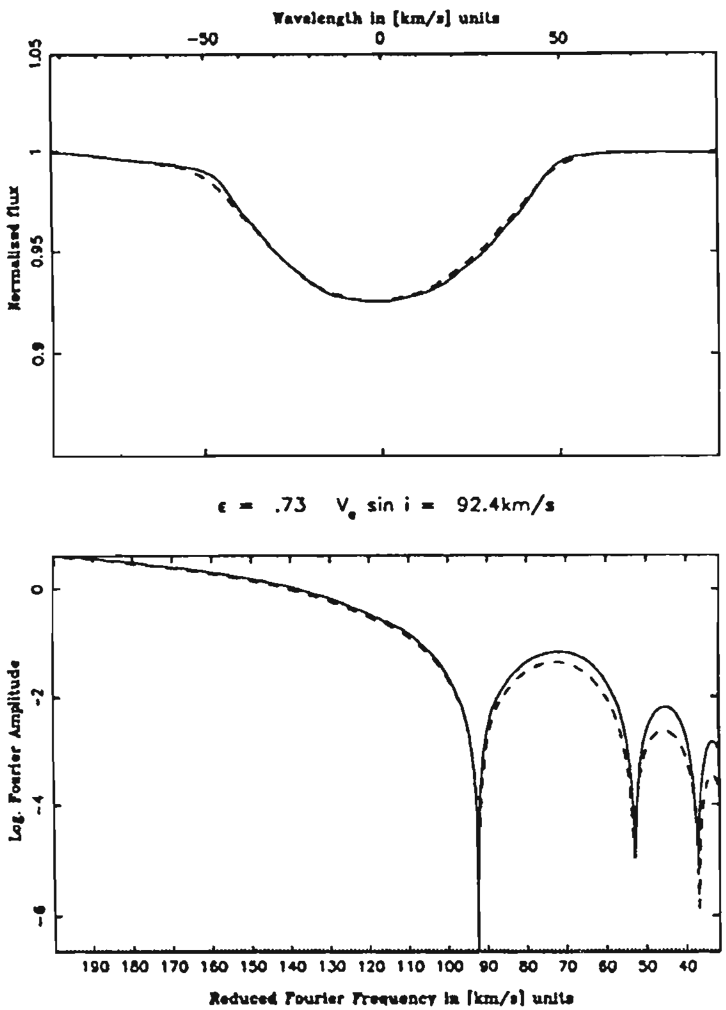

Fig. 2. **Top.** Full line represents the profile calculated under same conditions as on the Fig. 1. Dashed line shows the profile calculated performing disk integrations and using the model of radial-tangential macroturbulence of 5 kms^{-1}. **Bottom.** Fourier transforms of corresponding profiles are shown. Note that high Fourier frequencies of the profile represented by the dashed line are filtered by the macroturbulence.

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